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ABSTRACT

This guide is designed to aid the teacher in planning and teaching an eighth-grade mathematics course which should strengthen the student's understanding of the basic structure of mathematics through experience with and appreciation of abstract concepts. Thirteen units outlined are entitled: Numeration Systems; Natural Numbers and Zero; Integers; Equations; Writing and Solving Equations; Factoring and Prime Numbers; Rational Numbers; Ratio-Proportion-Percent; Points, Lines and Planes; Measurement; Construction; Perimeter-Area-Volume; and Linear Metric Measure. Behavioral objectives are listed for each unit. Aims, suggested materials and methods, and teaching techniques are written for each lesson. (JP)

DE SOTO PARISH CURRICULUM GUIDE

ED 092410

MATHEMATICS

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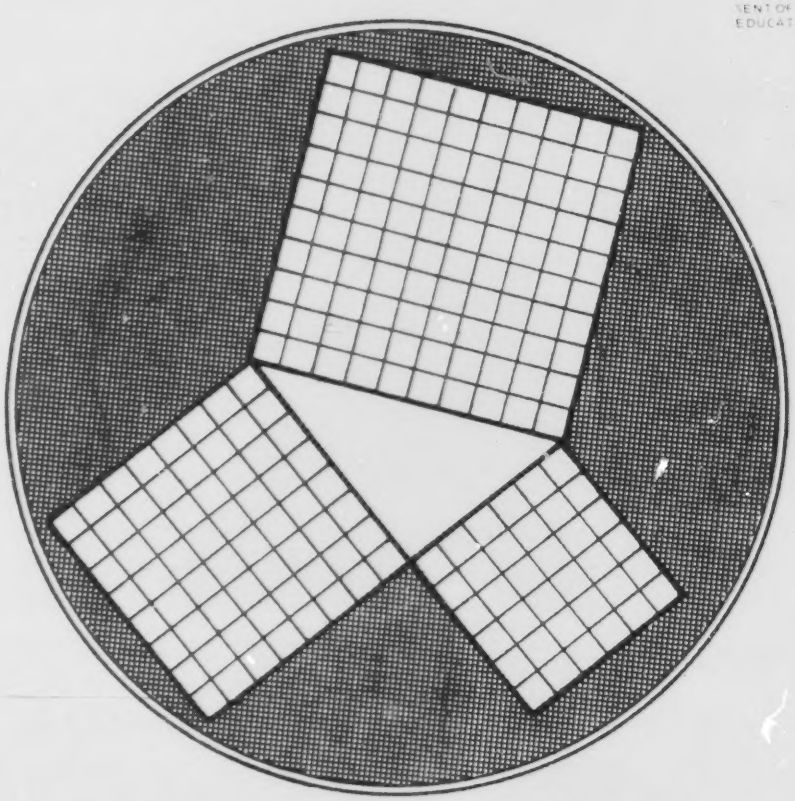

GRADE 8

SE 018 066

ED 092410

DE SOTO PARISH CURRICULUM GUIDE

Mathematics

Grade 8

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Issued by

DeSoto Parish School Board
Title I E.S.E.A.
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INTRODUCTION

The eighth grade is the last year that all the students take the same mathematics regardless of their abilities to deal with mathematical situations. The course is designed to strengthen the student's understanding of the basic structure of mathematics through experience with and appreciation of abstract concepts. It should prepare better students for advanced mathematics as well as strengthen weaker students for further study in general mathematics.

This guide is designed to aid the teacher in planning and teaching eighth grade mathematics. The committee suggests that it will be used as a flexible guide, keeping each individual class in mind.

The following suggestions might be considered in following and using this guide.

1. The committee suggests that if the majority of a class does not understand a particular lesson, the lesson should have additional work devoted to it before continuing with subsequent lessons.
2. Time allotments as given may vary with different classes and situations. Some of the lessons may require more than the allotted time. Also, no allotment has been made for a calculated twenty-five to thirty days to take care of unscheduled daily or weekly tests and days that the class may not meet.
3. There should be daily and/or weekly tests given; their time and content to be decided by each instructor.
4. A supplementary lesson or lessons could be added on averaging grades, using each instructor's grading scale and method, or the generally accepted method as a key.

5. Mathematical terms may be used as spelling words, in this way familiarizing the students with the words as well as their meanings.

6. Emphasis should be placed on neatness and accuracy of work and elimination of careless errors.

Mathematics 8

Scope, Sequence, and Time

Unit Topic	Text Pages	Time in Days
I. Numeration Systems (Chapter 1)		10
A. Number and Numeral	5	
B. The Decimal System	7	
1. Place Value	8-9	
2. Exponent Notation	8-9	
3. Other Bases to Show Place Value	10-19	
II. Natural Numbers and Zero (Chapter 2)		8-10
A. Define	23	
B. Set Notation	24	
C. One-to-One Correspondence	25	
D. Symbols of Grouping	26-27	
E. Closure	28-29	
F. Properties of Addition	30-31	
G. Properties of Multiplication	32-33	
H. Distributive Property	34-35	
I. Properties of One and Zero	38-39	
III. Integers (Chapter 3)		8-10
A. Number Line	43	
B. Graph of a Set of Numbers	44	
C. Addition and Multiplication of the Number Line	45	
D. Negative Integers	46	
E. Opposites	47	

Unit Topic	Text Pages	Time in Days
F. Addition	48-49	
G. Subtraction	50-51	
H. Multiplication	52-53	
I. Division	56	
J. Properties of the Operations with Integers	54-55	
IV. Equations (Chapter 4)		14-16
A. Math Sentences	61	
B. Order Relations	62-63	
C. Open Sentences	64	
D. Replacement Sets	65	
E. Subsets	66	
F. Disjoint Sets	67	
G. Solution Sets	68-69	
H. Equivalent Expressions	70-71	
I. Solving Equations	72-73	
1. Addition Property (one step equations)	74-75	
2. Multiplication Property (one step equations)	76	
3. Division Property (one step equations)	77	
4. Two-Step Equations	78-79	
V. Writing and Solving Equations (Chapter 5)		9-12
A. Open Expressions	83	
B. Translating English Phrases	84-85	
C. Open Sentences	86-87	

Unit	Topic	Text Pages	Time in Pages
	D. Solving Open Sentences	88-89	
	E. Use of Drawings to Help Solve Problems	90-91	
	F. Formulas as Equations	92-93	
	G. Estimating Answers	94	
VI.	Factoring and Prime Numbers (Chapter 6)		11-13
	A. Factors of Natural Numbers	99-100	
	B. Prime Numbers and Prime Factors	101-104	
	C. Divisibility	105-106	
	D. Review Exponent Notation	107	
	E. Greatest Common Factor	108-110	
	F. Relatively Prime Numbers	111	
	G. Least Common Multiple	112-113	
	H. Factors of Integers	114	
VII.	Rational Numbers (Chapter 7)		20-24
	A. Define (number line)	119-120	
	B. Equivalent Fractions	121-123	
	C. Multiplication Rational Numbers	124-125	
	D. Reciprocals	127	
	E. Division of Rational Numbers	128-129	
	F. Addition of Rational Numbers	130-132	
	G. Negative Rational Numbers	133	
	H. Subtraction of Rational Numbers	134	
	I. Distributive Property	135	
	J. Decimal Numerals	137	
	1. Add and Subtract Decimal Numbers	138-139	
	2. Multiply and Divide Decimal Numbers	140-141	

Unit	Topic	Text Pages	Time in Days
	3. Terminating Decimals	142	
	4. Repeating Decimals	143	
	5. Changing Decimals to Fractions	144	
	6. Order of Rational Numbers	145	
VIII.	Ratio-Proportion-Percent (skip to Chapter 12)		11-13 or 19-21
	A. Ratio	231-232	
	B. Rate	233 & 238	
	C. Proportion	234-235	
	D. Solving Proportions	236-238	
	E. Per Cent	239-242	
	F. Equivalent Fractions and Per Cent	242	
IX.	Points, Lines, and Planes (Chapter 8)		9-12
	A. Points and Lines	149	
	B. Subsets of a Line	150	
	C. Intersecting Lines	151	
	D. Planes	152-153	
	E. Parallel Lines and Planes	154-155	
	F. Polygons	156-157	
	G. Angles	158	
	1. Vertical and Adjacent	159	
	2. Measuring Angles	160-161	
	H. Perpendicular Lines and Planes	162-163	
X.	Measurement (Chapter 9)		10-12
	A. Precision	168	
	B. Greatest Possible Error	169	

Unit Topic	Text Pages	Time in Days
C. Decimal Notation	170	
D. Relative Error	171	
E. Significant Digits	172-173	
F. Adding and Subtracting Measures	174-175	
G. Multiplying and Dividing Measures	176-177	
XI. Construction (Chapter 10)		6
A. Congruent Line Segments	185	
B. Bisect Line Segments and Angles	186	
C. Construct Congruent Angles and Triangles	187-192	
D. Construct Perpendicular Lines	189	
XII. Perimeter-Area-Volume (Chapter 11)		17-20
A. Perimeter of Polygons	205	
B. Circumference of Circles	206-207	
C. Kinds of Quadrilaterals	208	
D. Area		
1. Rectangle	210	
2. Triangle	211-213	
3. Parallelogram	214	
4. Trapezoid	215	
5. Circle	216-217	
E. Volume		
1. Prisms	218-219	
2. Pyramid	220	
3. Cylinder		
a. Area	221	
b. Volume	222	

Unit Topic	Text Pages	Time in Days
4. Cone	224	
5. Sphere	226-227	
XIII. Linear Metric Measure	290-291	3-5

Goals

Eighth grade mathematics should:

1. Develop in the student the ability to work cooperatively with others.
2. Develop in the student the desire to acquire knowledge and achieve excellence.
3. Encourage an interest in and enjoyment of mathematics.
4. Provide the student with background knowledge and introduce new ideas necessary to continue study in later years.
5. Review mathematics concepts learned in earlier years.
6. Develop in the student the ability to use deductive reasoning.
7. Extend the student's ability to solve problems.
8. Review, clarify, and increase the mathematics vocabulary of the student.
9. Develop competence in computation.
10. Help the student express ideas accurately and effectively.
11. Help the student understand the basic structure of mathematics through experience with and appreciation of abstract ideas.
12. Develop ever-increasing proficiency in application of mathematics.
13. Develop in the student confidence in his ability to solve problems.
14. Develop in the student the ability to work neatly and to follow directions.

Unit I

Numeration Systems

Unit I

Numeration Systems

Behavioral Objectives:

1. Given numerals of a base, the student will give the place and face value of each digit in those numerals.
2. Given any power (as 3^2), the student will indicate the product as as single numeral.
3. Given any base ten numeral, the student will change that numeral to any other indicated base.
4. Given any base numeral, the student will change that numeral to a base ten numeral.
5. Given numerals of any base, the student will perform addition and multiplication.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Number and Numeral

Aim:

To help the students develop a better understanding of the distinction between number and numeral.

Suggested Material:

Picture of an object such as a car, dog or horse.

Suggested Methods:

Discussion, questions, and answers.

Developmental Steps:

1. Show a picture of something. Ask, "What is this?" When the student answers, say "No, this is a picture of a ...."
2. Put a numeral on the board (as 3). Ask, "What is this?" Explain that it is a numeral, not a number.
3. Use the illustrations on page 5 to show the number property of sets.
4. Read together and discuss the oral exercises on page 5.
5. Use the illustrations on page 6 and 7 to show grouping for numeration.
6. Discuss the word decimal and the prefix decimal.
7. Read together and discuss the oral exercise on page 7.
8. Demonstrate on the board how to work two or three examples similar to the written exercises on page 7.

Summary:

Review the meaning of number and numeral, grouping for numeration, and the word decimal.

Suggested Assignment:

Work the written exercises on page 7.

Lesson 2

Exponent Notation

Aim:

To teach the student to use exponent notation to illustrate an understanding of base ten numerals.

Suggested Methods:

Discussion, demonstration, and questions and answers on material on pages 8-9.

Visual Aid:

Transparency 511, Place Value

Developmental Steps:

1. Review the assignment by discussing and checking the work.
2. Discuss the meaning of the word digit.
3. Discuss the diagrams on page 8, showing place value of base ten numerals. (transparency 511)
4. Discuss the meaning of a power, using the examples on page 8.
5. Demonstrate on the board some examples of exponent notation.
6. Read together and discuss the oral exercises, page 9.
7. Ask the students to use exponent notation to show the face and place value of one or two decimal numerals.

Summary:

Review the meaning of digit, power, and exponent notation.

Suggested Assignment:

Work the written exercises on page 9.

Lesson 3

Base Five Numerals

Aim:

To teach the student about the base five numeration system to help him gain a deeper understanding of the decimal system.

Suggested Method:

Demonstration and drill on grouping in base five.

Developmental Steps:

1. Correct and review the assignment.
2. Read aloud the first paragraph on page 10.
3. Discuss the diagrams on page 10.
4. Have the students count off in the classroom in base five.
5. Draw a base five place value chart on the board, placing various numerals on the chart and showing how to use exponent notation to change them to base 10.
6. Discuss the oral exercises, page 11.
7. Work together in class problems 1a and 3a from the written exercises on page 11.

Summary:

Review place value in base five.

Suggested Assignment:

Work the written exercises on page 11.

Lesson 4

Changing Base Ten to Base Five

Aim:

To teach the students how to change a base ten numeral to a base five numeral.

Suggested Method:

Demonstration and discussion.

Supplementary Materials:

82 pennies, 3 quarters, 1 nickle

Developmental Steps:

1. Correct the assignment.
2. Demonstrate and explain the first paragraph on page 12 using the coins listed above.
3. Review how to use powers of five.
4. Demonstrate on the board the division method of changing base ten to base five.
5. Have the students change some numerals from base ten to base five.
6. Read together and discuss the oral problems on page 12.

Suggested Assignment:

Work the written exercises on page 12.

Lesson 5

Changing From Base Ten to Other Bases

Aim:

To teach the students to change base ten numerals to bases other than five.

Suggested Method:

Discovery, demonstration, discussion, and drill.

Developmental Steps:

1. Check and correct the assignment.
2. Say, "Suppose we wanted to change the number 5348 to base seven. What would we have to know?" (ans.: the powers of seven and the place value of base seven)
3. Refer students to the examples on page 13. Point out that the problems can be checked by changing the numeral back to base ten. Check an example to demonstrate what you mean.
4. Read, answer, and discuss "Oral", page 13.

Summary:

Work together problems 1c and 3c in the written exercises on page 13.

Suggested Assignment:

Work the remainder of the written problems on page 13.

Lesson 6

Adding Numbers in Different Bases

Aim:

To strengthen the students' understanding of addition in base ten by learning to add in other bases.

Suggested Methods:

Demonstration and discussion.

Developmental Steps:

1. Correct the assignment.
2. Discuss the diagrams in column one, page 14.
3. Call the students' attention to the addition chart on page 15, explaining its usage.
4. Develop an addition chart for base six on the board with students' help.
5. Add together some examples in different bases from the oral on page 15.

Summary:

Review the steps of addition. Emphasize that the concept of addition is the same regardless of the numeration system used.

Suggested Assignment:

Work the written problems on page 15.

Lesson 7

Multiplying Numbers in Different Bases

Aim:

To strengthen the students' understanding of multiplication in base ten by learning to multiply in other bases.

Suggested Methods:

Investigation and discovery.

Developmental Steps:

1. Check and correct the assignment.
2. Investigate the product of 3_{five} and 4_{five} using the illustrations on page 16.
3. Investigate the product of 23_{five} and 3_{five}, then look at the example on page 16.
4. How could we use this method on other bases?
5. Lead the student to discover that the concepts of multiplication are the same for any numeration system.

Summary:

Discuss the oral problems on page 17.

Suggested Assignment:

Work the written exercises, page 17.

Lesson 8

Base Two Numerals

Aim:

To review computation in other bases by working with base two numerals.

Suggested Methods:

Review and practice.

Developmental Steps:

1. Check and correct the assignment.
2. Review place value using base two.
3. Review the methods of changing other bases to base ten and of changing base ten to other bases by using base two.
4. Review addition and multiplication using base two.

Summary:

Discuss the oral problems on page 19.

Suggested Assignment:

Work the written problems on page 19.

Lesson 9

Review Unit I

Aim:

To review chapter 1 for a chapter test.

Suggested Methods:

Discussion, questions, and answers, and practice.

Developmental Steps:

1. Check and correct the assignment.
2. Using "Important Ideas," "Words to know" and "Questions to Discuss" on page 21, review the main ideas of the chapter.
3. Read and discuss the true or false statements on page 22.
4. Select and work together some problems from page 22 to review the methods previously taught.
5. Assign a chapter test for tomorrow. Suggest that the students study the chapter and work sample problems in preparation for the test.

Lesson 10

Chapter Test

Aim:

To test the students' behavior as stated by the behavioral objectives for Unit 1.

Suggested Problems:

1. Use exponent notation to show the face and place value of each digit in 302_{five}. _____ (2 problems)
2. 7^4 = _____ (1 problem)
3. Fill in the blanks (6 problems)

Base ten	Base five	Base two
72		
	32_{five}	
		11011_{two}

4. Use true or false statements from test, page T 22. (7 statements)
5. Use problems 1a, 1b, 3a, and 3b, page T 22.

Unit II

Natural Numbers and Zero

Unit II

Natural Numbers and Zero

Behavioral Objectives

1. Given any set of numbers, the student will properly define that set.
2. Given two sets, the student will determine one-to-one correspondence between those sets.
3. Given arithmetic equations involving parentheses and brackets, the student will solve those equations.
4. Given any set of numbers, the student will determine whether that set is closed under addition and multiplication.
5. Given any problem involving special properties of multiplication, addition, zero, or one, the student will identify those properties and correctly solve that problem.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Natural Numbers, Set Notation, and Correspondence Between Sets

Aim:

1. To develop within the student an understanding of the set of natural numbers.
2. To teach the student to properly denote and define various sets.
3. To develop the idea of one-to-one correspondence.

Supplementary Materials:

Filmloop No. MPL 510 - "One to One", filmloop projector

Suggested Methods:

Discussion, demonstration, and discovery.

Developmental Steps:

1. Return the tests; check and discuss them.
2. Define the set of natural numbers.
3. Lead the students to discover that each natural number is arrived at by adding one to the preceding number.
4. Discuss the oral exercise, page 23.
5. Discuss the material on page 24; answer orally the oral and the written exercises, page 24.
6. Discuss correspondence between sets. (Show filmloop no. MPL 510 at this point. Be sure you have previewed it and studied how it can best be discussed and used.)
7. Discuss the oral exercises on page 25.

Summary:

Define the set of natural numbers. Say again what must be done to define a set and what is meant by one-to-one correspondence.

Suggested Assignment:

Assign the written exercises, page 25.

Lesson 2

Symbols for Grouping

Aim:

To teach the students the need for and the use of parentheses and brackets as symbols for grouping.

Suggested Methods:

Comparison and demonstration.

Developmental Steps:

1. Use the examples on page 26 to explain the comparison of punctuation in English sentences to parentheses in mathematics sentences.
2. Use an example as (3 x 4 + 5) to explain how the meaning is changed in each grouping: (3 x 4) + 5 and 3 x (4 + 5). Explain the rule of order on page 26 when no parentheses are used.

Summary:

Use the oral problems to demonstrate and practice. (Emphasize form)

Suggested Problems:

Work problems 5-10 and 15-23 in the "Written," page 27.

Lesson 3

Closure

Aim:

To teach the students the meaning of closure and that the set of natural numbers is closed under addition and multiplication.

Suggested Methods:

Discussion, questions, and answers.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material in column one on page 28.
3. Define closure. (Remind the students to be sure to define a set and an operation before trying to determine closure.)
4. Use the set of natural numbers with each operation to determine closure. Ask, "Why is it not closed under subtraction and division?"
5. Use other example sets as (as $\{0, 1\}$); determine closure under various operations.

Summary:

Use the oral on page 29.

Suggested Problems:

Work the written exercises on page 29.

Lesson 4

Properties of Addition

Aim:

To teach the student to recognize and use the properties of addition.

Suggested Methods:

Discovery and definition.

Developmental Steps:

1. Check and correct the assignment.
2. Ask the students to add 3 numbers as (5, 8, and 6). Ask different ones to tell how they added the three numbers. (Put examples on the board and leave them there.)
3. Lead the students to discover that you can only add two numbers at a time. Define the word binary and binary operation.
4. Examine other operations to see if they are binary.
5. Go back to the examples on the board. Lead the students to discover that it doesn't matter what order of adding numbers is used, or which two are added first, the results are the same.
6. Define the communtative property of addition. Ask, "Which examples on the board show the commutative property?"
7. Define the associative property. Ask, "Which examples apply to this property?"

Summary:

Discuss the oral, page 31.

Suggested Problems:

Work the written exercises, page 31.

Lesson 5

Properties of Multiplication and the Distributive Property

Aim:

1. To teach the student that the properties of multiplication and addition are the same.
2. To teach the student to recognize and use the distributive property.

Suggested Methods:

Discussion, question, and answer, discovery, and drill.

Suggested Materials:

Transparencies 511, numbers 600-604, overhead projector

Developmental Steps:

1. Check and correct the assignment.
2. Use transparencies 601, 602, and 604.
3. Use several examples to examine multiplication to see if the commutative and associative properties of multiplication hold true.
4. Use the examples on page 33 to check subtraction and division to show that they are not associative and commutative.
5. Use the example about selling tickets on page 34 to introduce the distributive property. (Emphasize that it is the only property involving two operations.)
6. Use transparencies 511, numbers 602 and 603.
7. Use problems 1, 11, 16, 17 on page 35 to show how the distributive property works.

Summary:

Discuss the oral, page 35.

Suggested Problems:

Work problems 9-14, p. 33, and the remainder of the problems in the written exercise, page 35.

Lesson 6

Using the Properties of Operations and the Properties of One and Zero

Aim:

1. To strengthen the students' understanding of the properties of operations.
2. To develop an understanding of the properties of one and zero.

Suggested Methods:

Review, discussion, and discovery.

Developmental Steps:

1. Check and correct the assignment.
2. To review, use the examples on page 36.
3. Discuss the oral on page 36.
4. Do together examples 1 and 4 in the written exercise, page 37.
5. Discuss the material in the left column, page 38. Multiply 1 by several numbers. Lead the students to discover and state the identity property of one.
6. Review exponent notation. Lead them to discover that one to any power is equal to one.
7. Use the examples in the right column to lead them to discover that one is the identity of division also.
8. Lead the students to discover that zero is the identity element of addition and subtraction.
9. Discuss the multiplication property of zero.
10. Try division by zero, leading them to discover that it is meaningless.

Summary:

Discuss the oral exercises, pages 38 and 39.

Suggested Problems:

Work examples 2, 3, and 7-12 from the written exercises, pages 38-39.

Lesson 7

Review Unit II

Aim:

To review chapter II for chapter test.

Suggested Methods:

Discussion, question, and answer, and practice.

Developmental Steps:

1. Check and correct the assignment.
2. Using "Important Ideas," "Words to Know" and "Questions to Discuss," page 41, review the main ideas of the chapter.
3. Read and discuss the true or false statements on page 42.
4. Select and work together problems from page 42 to review the various types.

Suggested Assignment:

Assign the chapter test for tomorrow. Suggest that the student study the chapter and work sample problems in preparation for the test.

Lesson 8

Chapter Test

Aim:

To test the students' behavior as stated in the behavioral objectives.

Suggested Materials:

Mimeographed test

Suggested Problems:

1. $\{3, 6, 9, 12...\}$ Define this set. (2 problems)
2. Write this set: The set of natural numbers less than 6. (2 problems)
3. Name a set that is in one-to-one correspondence with the set of months of the year. (2 problems)
4. Name N as a single interger. $72 - (45 \div 9) + 32 = N$

   $(3 + 4) \times 5 = N$ (5 problems)
5. Fill the blanks to tell whether each set is closed under each operation. (4 problems)

	Add	Sub.	Mult.	Div.
$\{0, 1\}$				
$\{1, 2, 3, 4\}$				

6. Examples like Part 2, page T 42. (10 problems)

Unit III

Integers

Unit III

Integers

Behavioral Objectives:

1. Given any set of numbers, the students will graph the set correctly on a number line.
2. Given any addition or multiplication example, the students will show that operation on the number line.
3. Given an integer, the students will indicate the opposite of that integer.
4. Given any integers, the student will add, subtract, multiply, or divide correctly by formulated rules.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

The Number Line

Aim:

To teach the students how to construct a number line, give the coordinates of the points associated with zero and the natural numbers, and graph a set of numbers.

Suggested Methods:

Demonstration, discussion, and investigation.

Developmental Steps:

1. Return the chapter test and discuss it. Have the students correct their errors.
2. Demonstrate the number line by drawing one on the board, putting two points on it, and labeling them 0 and 1.
3. How will the location for the point 2 on the number line be found? Emphasize that the point 1 must be equidistant from 0 and 2.
4. Discuss the first paragraph, right column, or page 43. Bring in one-to-one correspondence between points and coordinates.
5. Have each student fold a piece of paper lengthwise two times to use to draw number lines. The lines on the paper can be used to measure distance between points.
6. Discuss the graph of a set using the material in the first column on page 44.
7. Demonstrate the graph of a set on the board.

Summary:

Use the oral, pages 43 and 44.

Suggested Problems:

Work the written exercises, page 44.

Lesson 2

The Set of Integers

Aim:

1. To teach the students to interpret addition and multiplication from a geometric point of view.
2. To introduce the set of integers.

Suggested Methods:

Demonstration, discovery, and discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Draw number lines and demonstrate addition and multiplication as repeated addition on the number line. Emphasize direction.
3. Discuss the oral exercise, page 45.
4. Draw a number line on the board extending to the left of zero. Ask, "What would we have to the left of zero?" Lead the student to discover the negative integers and order on the number line.
5. Define the set of integers as made up of three parts; the positive numbers, zero, and the negative numbers.
6. Use the material in column one, page 47, to introduce opposites.

Summary:

Discuss the oral exercises, page 47.

Suggested Problems:

Work the written exercises, page 45, numbers 1 and 2; all the written, page 47.

Lesson 3

Addition of Integers

Aim:

To teach the concepts of addition of integers.

Suggested Material:

Filmloop no. MPL 510, Directed Numbers, filmloop projector

Suggested Methods:

Demonstration, discovery, and drill.

Developmental Steps:

1. Check and correct the assignment.
2. Use the filmloop. Preview it and be ready to use it as it directs.
3. Demonstrate addition of integers on the number line. Emphasize positive and negative direction. (Leave the diagrams on the board.)
4. Using these examples, lead the students to discover the rules for adding integers.
5. Drill on addition using problems 1-8 in the written, page 49.

Summary:

Discuss the oral exercises, page 49.

Suggested Problems:

Work problems 9-16 in the written exercises, page 49.

Lesson 4

Subtraction of Integers

Aim:

To teach the concepts of subtraction of integers.

Suggested Methods:

Review, discussion, question and answer.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss addition and subtraction as inverse operations. Use examples from page 50 with the number line.
3. Use examples at the top of the first column, page 51, to derive the rule for subtraction.
4. Show examples on the board of changing subtraction to addition. (like 2a, 2b, 4a, and 4b in the written exercises)

Summary:

Discuss the oral exercises, pages 50 and 51.

Suggested Problems:

Work the written exercises, page 51.

Lesson 5

Multiplication of Integers

Aim:

To teach the concepts of multiplication of integers.

Suggested Methods:

Discovery, discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Read together and discuss the material on page 52 and the first column on page 53.
3. Restate the rules for multiplying integers.
4. Do the oral, page 53, and problems 1-2 of the written, together.

Summary:

Restate the rules for multiplication of integers.

Suggested Problems:

Work problems 3-11 in the written exercises, page 53.

Lesson 6

The Properties With Respect to the Set of Integers

Aim:

To strengthen the students' understanding of multiplication, subtraction, and addition through investigation of the various properties with respect to the set of integers.

Suggested Methods:

Investigation, deduction, and discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Investigate the properties as outlined on pages 54-55. Use suggested questions to draw conclusions.
3. Emphasize: a. That the set of integers is closed under subtraction.
   b. That zero and one are still the identity elements.
4. Discuss the oral, page 55.
5. Work together problems 1, 6, and 11 in the written, page 55.

Summary:

Restate the properties with respect to the set of integers.

Suggested Problems:

Work the remainder of the written, page 55.

Lesson 7

Division of Integers

Aim:

To teach the concepts of division of integers.

Suggested Methods:

Discovery and discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss multiplication and division as inverse operations.
3. Use examples a, b, and c, page 56, to lead the students to discover the division rules.
4. Discuss the oral, page 56.
5. Use the rules to divide in problems 1-4 in the written, page 56.

Summary:

Restate the rules for division of integers.

Suggested Problems:

Work problems 5-10 in the written, page 56.

Lesson 8

Using Integers and Chapter Review

Aim:

1. To teach the use of positive and negative integers in solving problems.
2. To review for chapter test.

Suggested Methods:

Discussion and review.

Developmental Steps:

1. Check and correct the assignment.
2. Use several examples of things to which positive and negative integers can be assigned (upstream-downstream, gain-loss, etc.).
3. Discuss problems on page 57 as suggested by the oral.
4. Discuss the review material, page 59.
5. Work together some examples (page 60) to review rules and methods.

Summary:

Review rules for addition, subtraction, multiplication, and division.

Suggested Problems:

Work the written exercises, page 57; part 4, page 60. Assign the chapter test for tomorrow.

Lesson 9

Chapter Test

Aim:

To test the students' behaviors as stated in the behavioral objectives.

Suggested Methods:

Testing

Supplementary Materials:

Mimeographed tests

Developmental Steps:

1. Check and correct the assignment. Answer any questions.
2. Give the test.

Suggested Problems:

1. Graph this set: $\{-3, -4, 0, 3, 4\}$
2. Draw a number line and show the following: $5 + (-2)$
3. Use part 3 and part 4 of chapter test on page T 60.

Unit IV

Equations

Unit IV

Equations

Behavioral Objectives:

1. The students will show their understanding of order relations, open sentences, replacement sets, subsets, disjoint sets, solution sets, and equivalent expressions by indicating whether mathematical sentences and/or statements involving them are true or false.
2. Given simple equations involving one variable, the students will solve those equations using the addition, multiplication, and division properties of equations.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Sentences About Numbers and Order Relations

Aim:

To help the student review the meaning of the = and ≠ signs, and to develop the concept of less than and greater than in order relations.

Suggested Methods:

Demonstration, discussion, and drill.

Developmental Steps:

1. Return and discuss the chapter test. Let the students correct their errors.
2. Discuss the examples on page 61.
3. Make up examples which will show equality and inequality.
4. Discuss the oral exercise on page 61.
5. Discuss the material on pages 62 and 63. Use the number line to stress ordering numbers.
6. Discuss the meaning of $>$ and $<$.
7. Demonstrate less than and greater than on the number line. Emphasize that the number whose point is the farthest to the right on the number line is the larger number.
8. Work examples 1, 5, and 8 together under written on page 63.

Summary:

Use the oral exercise, page 63.

Suggested Assignment:

Work the remainder of the written examples, page 63.

Lesson 2

Open Sentences and Replacement Sets

Aim:

To develop within the student the concept of open sentences and replacement sets.

Suggested Methods:

Discussion, demonstration, and discovery.

Developmental Steps:

1. Check and correct the home assignment.
2. Discuss the material on page 64. Define: open sentence, variable.
3. Demonstrate the use of open sentences.
4. Discuss the oral exercises on page 64.
5. Work and discuss the written exercises on page 64.
6. Discuss the material on page 65. Emphasize the meaning of replacement set.
7. Work the examples on page 65.
8. Work and discuss examples 1 and 6 together under written, page 65.

Summary:

Use the oral exercises on page 65.

Suggested Assignment:

Work the written exercises on page 65.

Lesson 3

Subsets

Aim:

To develop within the student an understanding of subsets.

Suggested Methods:

Discussion and demonstration.

Developmental Steps:

1. Check and correct the home assignment.
2. Discuss the material on page 66. Define subset. Use some examples of sets. Determine if one of the sets is a subset of another.
3. Illustrate some subsets by using the students and the material in the classroom.
4. Do one example under the written exercise.

Summary:

Do the oral problems on page 66.

Suggested Assignment:

Work the written exercises on page 66.

Suggested Quiz:

1. Use 3 examples like those on page T 63.
2. Use 2 examples like those on page T 65.

Give the quiz. Discuss errors that occur.

Lesson 4

Disjoint Sets

Aim:

To help the student develop an understanding of disjoint sets.

Suggested Methods:

Discussion, discovery, and demonstration.

Suggested Materials:

Transparencies with overlays to demonstrate Venn diagrams the intersections or common elements of two sets, and disjoint sets; overhead projector

Developmental Steps:

1. Check and correct the home assignment.
2. Consider two sets such as A = $\{2, 5, 6, 7, 9\}$; B = $\{1, 2, 6, 7\}$. Discuss their common elements, the elements of A that are not in B; the elements of B that are not in A. Use the transparencies and discuss Venn diagrams, disjoint sets, and the intersection of sets (common elements).
3. Question the students to help them discover why sets are called disjoint.
4. Discuss examples 1 and 5 in the written exercise on page 67.

Summary:

Use the oral exercise on page 67.

Suggested Assignment:

Work the written exercise on page 67.

Lesson 5

Solution Sets

Aim:

To help the students develop an understanding of the Solution Sets.

Suggested Methods:

Discussion and demonstration.

Developmental Steps:

1. Check and correct the home assignment.
2. Discuss the material on pages 68 and 69. Emphasize that the set of elements of the replacement set which makes the sentence true is called the solution set. The set of elements which makes the sentence false is disjoint from the solution or "truth" set.
3. Demonstrate the Solution Sets by graphing them on the number line. Discuss the empty set and its symbol.
4. Discuss examples 1, 5, and 11 in the written exercise on page 69.

Summary:

Use the oral exercise on page 69.

Suggested Assignment:

Work the written exercise on page 69.

Lesson 6

Equivalent Expressions

Aim:

To teach the student the meaning of equivalent expressions.

Suggested Methods:

Discussion, review, demonstration, and discovery.

Developmental Steps:

1. Check and correct the assignment.
2. Develop through discussion what is meant by mathematical expressions.
3. Use the examples on page 70 to help the students understand and prove that $3Y + 4Y = 7Y$ for any value of Y. These are called equivalent expressions as they have the same value.

$$\begin{aligned} &3y + 4y = \\ &(y \times 3) + (y \times 4) = \\ &y\ (3 + 4) = \\ &7y \end{aligned}$$

Relate equivalent expressions to the distributive property. (Review the distributive property.)

4. Discuss the oral exercise on page 70.
5. Develop the property of negative one. Discuss the proof and examples on page 71. Encourage the students to justify each step in the proofs and examples.

Summary:

Use the oral exercise on page 71.

Suggested Assignment:

Work the written exercises, page 71.

Lesson 7

Solving Equations

Aim:

1. To develop the idea of the equation as a special kind of mathematical sentence.
2. To develop the idea of the root of an equation as the solution set.

Suggested Methods:

Discussion, demonstration, and drill.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the different kinds of sentences. Develop the idea that equations may be true, false, or open sentences. The equations we will be working with are open sentences and we will be trying to find the truth set.
3. Introduce the term root and define it as the solution set of an equation. Stress that our replacement set, if not specified, is the set of integers.
4. Use examples, page 72 and 73, to develop methods of solving equations by trial and error.
5. Use examples 1, 2, 11, and 12 as drill on solving equations.

Summary:

Discuss the oral exercises, page 73.

Suggested Assignment:

Work the remainder of the written exercises, page 73.

Lesson 8

Addition Property of Equations

Aim:

To teach the students to use the addition property of equations.

Suggested Methods:

Discussion, demonstration, and drill.

Developmental Steps:

1. Check and correct the assignment.

2. Discuss the material on page 74 to develop the addition property.

$$
\begin{aligned}
&y + 5 = 20 \\
&(y + 5) + -5 = 20 + (-5) && \text{Why?} \\
&y + [5 + (-5)] = 20 + (-5) && \text{Why?} \\
&y + 0 = 15 && \text{Why?} \\
&y = 15
\end{aligned}
$$

Optional 3. Introduce these forms:

$$
\begin{array}{r}
A + 6 = 11 \\
-\ 6 = -6 \\
\hline
A + 0 = 5 \\
A = 5
\end{array}
\qquad \text{Check } 5 + 6 = 11
$$

$$
\begin{aligned}
A - 10 &= -5 \\
A + (-10) &= -5 \\
A + [(-10 + 10)] &= -5 + (10) \\
A + 0 &= 5 \\
A &= 5
\end{aligned}
$$

This is a good time to introduce checking. (Substitute 5 for A.)

$$
\begin{aligned}
5 - 10 &= -5 \\
-5 &= -5
\end{aligned}
$$

Show that this is the addition property of equality.

4. Use examples 1-3, page 75. Show both methods of solving these equations.

5. Discuss the checking of equations.

6. Work together exercises 1-4 in the written exercises.

Summary:

Discuss the oral exercises, page 75.

Suggested Assignment:

Work the remainder of the written exercises, page 75.

Lesson 9

Multiplication Property of Equations

Aim:

To teach the students to use the multiplication property of equations.

Suggested Methods:

Discussion, testing, drill.

Developmental Steps:

1. Check and correct the assignment.
2. Give a quiz on the addition property (as: $n + 5 = 6$; $11 = a - 3$; $34 = r - 46$; about two of each type).
3. Read together and discuss the material, page 76, to introduce the multiplication property. Work one or more examples.

Optional 4. Show this form: $\frac{Y}{2} = 6$

$$2 \times \frac{Y}{2} = 6 \times 2$$

$$Y = 12$$

This form may be used: $2\left(\frac{4}{2}\right) = 2(6)$

Explain that this the multiplication property of equations.

5. Work together examples 1a and 9a.

Summary:

Discuss the oral exercises, page 76.

Suggested Assignment:

Work the remainder of the written exercises, page 76.

Lesson 10

Division Property of Equations

Aim:

To teach the student to use the division property of equations.

Suggested Methods:

Discussion, testing, drill.

Developmental Steps:

1. Return the test and let the students correct their errors.
2. Check and correct the assignment.
3. Give a quiz on the multiplication property. Use examples like the following:

   $\frac{N}{7} = 7$, $-12 = \frac{A}{-3}$, $\frac{B}{3} = -11$ (about 2 of each)

4. Discuss the material, page 77, to introduce the division property of equations. Use an example such as $3x = 12$ to show the division property. State it in the generalized form.
5. Discuss examples 1-3.
6. Use the division property and the properties of operations to justify each step of the solutions of the following. Work together examples 1a and 8a.

Summary:

Do the oral exercises, page 77.

Suggested Assignment:

Work the remainder of the written exercises, page 77.

Lesson 11

Using the Properties of Equations

Aim:

To introduce to the student equations involving more than one operation.

Suggested Methods:

Demonstration, discussion, and review.

Developmental Steps:

1. Return the tests and correct errors as the papers are discussed.
2. Check and correct the assignment.
3. Review the properties of equations. Solve an equation, telling the properties that justify each step.
4. Use the examples on page 78 to illustrate different kinds of equations.
5. Have the students work and correct number 3 and 4 in the oral exercise and 1a, 1b, 6b, and 9b in the written exercise, page 79. Do these one at a time, correcting each before working the next one.

Summary:

1. Review the properties (1, 2, oral).
2. Discuss the steps to go through in solving the problems in the written exercises.

Suggested Assignment:

Work the remainder of the written, page 79.

Lesson 12

Using the Properties of Equations (Continued)

Aim:

To continue solving equations involving more than one step.

Suggested Methods:

Developmental Steps:

1. Discuss, review, and correct all examples from the assignment.
2. Select an equation and solve it with student assistance.

Summary:

Discuss different kinds of equations.

Suggested Assignment:

Have the students solve examples 11-19, page 73. Tell them that these were not solved before. The solutions were found by trial and error.

Lesson 13

Chapter Review

Aim:

To review for the chapter test.

Suggested Methods:

Review, discussion.

Developmental Steps:

1. Check and discuss the assignment.
2. Using pages 81 and 82 as a guide, review the chapter, working sample problems and reviewing the properties of equations, etc.

Suggested Assignment:

Chapter Test Tomorrow.

Lesson 14

Chapter Test

Aim:

To test the students' behaviors as stated in the behavioral objectives for Unit IV.

Suggested Methods:

Testing

Developmental Steps:

Give the chapter test as given on page T 82 except for part 4. Instead, give one-step equations as:

1. $Y + 6 = 28$
2. $56 = r - 27$
3. $T + (-3) = 7$
4. $\frac{C}{8} = 9$
5. $15 = \frac{n}{4}$
6. $\frac{C}{3-2} = 16$
7. $2n = 18$
8. $-51 = -3n$
9. $n(5 - 9) = 0$

Select two from Part 4 page T 82.

Unit V

Writing and Solving Equations

Unit V

Writing and Solving Equations

Behavioral Objectives:

1. Given English phrases and sentences, the student will translate those phrases and sentences into mathematical expressions or sentences. (and vice-versa)
2. Given problems to solve, the student will demonstrate the ability to estimate and solve those problems.
3. The student will demonstrate his understanding of the relationship between equations and formulas by solving problems using the necessary formulas.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Open Expressions

Aim:

To teach the student to translate open mathematical phrases into English phrases.

Suggested Methods:

Review and discussion.

Developmental Steps:

1. Return the chapter test. Discuss them. Allow the students to correct errors as the test is discussed.
2. Discuss what is meant by the term open expression.
3. Discuss the material in the first column and the chart on page 83.
4. Give some examples of open expressions and what they might mean. Encourage student participation in translating the open expressions.

Summary:

Discuss the answers to the questions in the oral exercises, page 83.

Suggested Assignment:

Work the written exercises, page 83.

Lesson 2

Translating English Phrases

Aim:

To teach the student to translate English phrases into mathematical phrases.

Suggested Methods:

Discussion, discovery.

Developmental Steps:

1. Check and correct the assignment.
2. Use the examples on page 84 to discuss translating English phrases to mathematical expressions.
3. Pass out mimeographed sheets on simple expressions (attached). Do these together in class.
4. Discuss the oral exercises, page 85.

Summary:

Discuss equivalent English words and operation symbols as given on page 84. Suggest others.

Suggested Assignment:

Work the written exercises, page 85.

If p is a certain number, then write the expressions for:

1. 3 more than twice the number
2. 3 times the number
3. the sum of the number and 6
4. the number increased by 3
5. 6 less than the number
6. the number decreased by 6 and then the result doubled
7. the product of twice the number and 14
8. 1/2 of three times the number
9. 6 less than 5 times the number
10. the sum of the number and twice the number

Lesson 3

Open Sentences

Aim:

To teach the student to translate English sentences into mathematical sentences.

Suggested Methods:

Discovery, discussion, comparison.

Developmental Steps:

1. Correct and discuss the assignment.
2. Discuss the uses of English expressions and sentences. Compare these with the uses of mathematical translations. Discover the relation between the verb of the sentence and the equality sign. Take an example of an English sentence, choose a symbol to represent the variable and translate from the English sentence to a mathematical sentence.
3. Read and discuss the material on page 86.
4. Take some of the expressions in the assignment and discuss how they might have been completed as sentences.

Summary:

Discuss the oral exercises, page 87, as a class activity.

Suggested Assignment:

Work the written exercises, page 87.

Lesson 4

Solving Problems

Aim:

To develop the students' ability and skill in problem solving by using equations in the solutions.

Suggested Methods:

Discussion, demonstration.

Suggested Materials:

Transparency of the suggestions for problem solving, a-d, page 88, overhead projector

Developmental Steps:

1. Check and correct the assignment.
2. Emphasize that there is no single set of rules to apply to solve problems.
3. Go over the steps suggested in the first column on page 88. Use the transparency of the steps for problem solving.
4. Use example problems on page 88 following the suggested steps. Be sure to let the students justify each step.
5. Using the sample problem, page 89, discuss the three translations given.
6. Discuss and answer the oral questions. Emphasize that there may be different translations that are correct.

Summary:

Read together and discuss possible translations of as many of the written problems as possible, page 89.

Suggested Assignment:

Work the written exercises, page 89. Be sure to state what the variable represents as well as to solve the problems.

Lesson 5

Using Drawings to Help Solve Problems

Aim:

1. To introduce the idea of using drawings to help understand the situation of problem solving.
2. To continue practice on problem solving.

Suggested Methods:

Discussion and practice.

Developmental Steps:

1. Check, discuss, and correct the assignment.
2. Use examples to show how drawings can help the student understand problems. Let the students justify the steps.
3. Look at the various problems in the written to see which ones would lend themselves to drawings and which ones would not. Have students sketch one or two of the problems and discuss them.

Summary:

Do the oral, page 91.

Suggested Assignment:

It is suggested that some of these problems may be too difficult for some students. An average class could be assigned problems 2, 3, 4, 5, 7, and 8. Encourage the better students to solve the others.

Lesson 6

Using Formulas to Solve Problems

Aim:

To teach the student to use formulas to solve problems.

Suggested Methods:

Discussion, demonstration.

Developmental Steps:

1. Check, discuss, and correct the assignment.
2. Define: formula, rate, distance, time, interest, area, etc. Discuss the examples on pages 92-93 or other problems that involve the various formulas.
3. Discuss some of the problems in the written, suggesting or having students suggest formulas that may be used to help solve them.

Summary:

Discuss the oral exercise, page 93. Discuss other formulas in the same vein.

Suggested Assignment:

It is suggested that problems 5 and 6 may be too difficult for some students. It is suggested that an average class might be assigned problems 1, 2, 3, 4, 7, 8, and 9, page 93.

Lesson 7

Estimating Answers

Aim:

To teach the student to use an estimated answer (an approximate answer) as a check on computation.

Suggested Methods:

Discussion, demonstration, practice.

Developmental Steps:

1. Check and correct the assignment.
2. Read and discuss the material in the first column, page 94.
3. Review problems from the assignment and discuss how answers might be estimated. Emphasize that answers should always be checked to see if they are reasonable.
4. Use the oral exercises to practice estimating.

Summary:

Review the steps used in estimating answers.

Suggested Assignment:

It is suggested that many of these problems are too difficult for some students. It is suggested that an average class might be assigned problems 1-3, page 94, and problems 1, 6, 7, 10, 11, page 95.

Lesson 8

Chapter Review

Aim:

To review for the chapter test.

Suggested Methods:

Discussion, review.

Developmental Steps:

1. Check and correct the assignment.
2. Using pages 97-98 as a guide, review the material covered in the chapter. Work sample problems together.

Suggested Assignment:

Chapter test will be given tomorrow.

Lesson 9

Chapter Test

Aim:

To test the students' behavior as stated in the behavioral objectives for Unit V.

Suggested Methods:

Testing

Supplementary Materials:

Copies of the test

Developmental Steps:

Use the following parts of the chapter test as found page T 98.

Part 1 - 1a and 2a

Part 2 - 1-4

Part 4 - 1, 2 (Have students write equations and solve problems.)

Add two or more written problems similar to those assigned daily.

It is suggested that more point value should be given for Part 4 problems than for Parts 1 and 2.

Unit VI

Factoring and Prime Numbers

Unit VI

Factoring and Prime Numbers

Behavioral Objectives:

1. The student will demonstrate an understanding of the terms factor, prime number, composite number, divisible by, multiple of, greatest common factor, least common factor, and least common multiple by defining them and/or by determining whether statements concerning them are true or false.
2. Given a numeral, the student will find its prime factors.
3. Given a set of numbers, the student will find the greatest common factor and least common multiple of that set.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Factors of Natural Numbers

Aim:

To teach the student to factor a number to its smallest factors.

Suggested Methods:

Discussion, demonstration.

Suggested Materials:

Filmloop MPL 511, Sets of Factors, filmloop projector

Developmental Steps:

1. Return the tests. Discuss them. Give the students time to correct errors they have made as the test is discussed.
2. Define the word factor as suggested on page 99.
3. List with student help all of the factors of 12.
4. List all of the factors of 84. Send a student to the board to do this. Allow other students to suggest other factors of 84.
5. Use these to name 84 in factored form in as many different ways as possible (as the product of two factors). Leave it on the board.
6. Now go back and begin factoring to the smallest factors using those left on the board. Have students do some of them.
7. Show the filmloop. Be sure to use it as directed for best results.
8. Discuss what is meant by "numbers in their natural order."
9. Emphasize that no matter in what order a number is factored, the smallest (or prime) factors will be the same.

Summary:

Use the oral exercises as a class activity.

Suggested Assignment:

Work the written exercises, page 100.

Lesson 2

Prime Numbers

Aim:

To develop in the student an understanding of prime and composite numbers.

Suggested Methods:

Discussion, demonstration, discovery.

Suggested Materials:

Transparency of the whole numbers named in their natural order from 1 to 100, to be used in demonstrating the Sieve of Erathosthenes

Developmental Steps:

1. Check and correct the assignment.
2. Using the material on page 101, define prime and composite numbers. Have the students name the prime numbers smaller than 50.
3. Lead the student to discover that the smallest factors of a number are the prime factors. Set up several numbers and factor them with student help.
4. Demonstrate (using numbers) how the Sieve of Eratosthenes works to find prime numbers. (page 102, text; or page 84, Mathematics 7, Laidlaw series) Use the transparency to demonstrate this.
5. Discuss briefly Goldbach's conjecture and the "Fundamental Theorem of Arithmetic." (page 104)

Summary:

1. Define prime and composite numbers again.
2. Use the oral, page 104.

Suggested Assginment:

Work the written exercises, page 104, lines 1 and 2.

Lesson 3

Divisibility

Aim:

To teach the students the rules for divisibility.

Suggested Methods:

Discussion, demonstration, drill.

Developmental Steps:

1. Check and correct the assignment.
2. Define the words divisible by as opposed to can be divided by.
3. Read and discuss rules for divisibility by 2, 3, and 5. (page 105)
4. Work several examples from the written, page 105, to check the use of these rules.
5. Read and discuss the rules for divisibility by 6, 7, and 10. Show several examples of each. Develop a rule to test divisibility by 9. Let the students help.

Summary:

Use oral exercise:

1. Extend No. 1 to include other numbers as 15.
2. Extend number 2 to include divisibility by 2, 3, and 5.

Suggested Assignment:

Work the written, page 106, lines 3 and 4. Have the students first list the numbers by which each is divisible before factoring.

Lesson 4

Using Exponents

Aim:

To teach the student to use exponents to state prime factors.

Suggested Methods:

Discussion, drill, and review.

Suggested Materials:

Prepared transparency like the diagram on page 107, overhead projector

Developmental Steps:

1. Check and correct the assignment.
2. Review the meanings of power, base, exponent. Use a prepared transparency like the diagram in the text as you recall the meanings of the words.
3. Use examples 2^3, 4^2, 5^7, 8^4. Have students name the base and exponent in each and tell the decimal value of each power.
4. Discuss some examples which use exponents in stating the prime factors of numbers, as $20 = 2 \times 2 \times 5 = 2^2 \times 5$.

Summary:

Write some of the factors in yesterday's assignment using exponent notation.

Suggested Assignment:

Do the oral, lines 3 and 4 and the written, lines 1 and 3, page 107.

Lesson 5

Greatest Common Factor

Aim:

To teach the student how to find the greatest common factor of several numbers using a complete factorization of each number.

Suggested Methods:

Discussion and demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Define the term, "greatest common factor."
3. List together all the factors of two numbers such as 16 and 24. Identify which factors are common factors. Then ask, "Which is the greatest common factor."
4. Discuss the material on page 108.
5. Discuss the steps used in determining the greatest common factor. List them on the board.
6. Discuss some examples on page 109. Have the students help with finding the prime factors and the greatest common factor.
7. Work together and discuss 1a, 1b, and 4b in the written assignment.

Summary:

Use the oral, page 109. Review the steps for finding the greatest common factor.

Suggested Assignment:

Work the remainder of the written, page 109.

Lesson 6

Great Common Factor (Con't.)

Aim:

To teach the students a method for finding the greatest common factor of two large natural numbers.

Suggested Methods:

Discussion, discovery.

Developmental Steps:

1. Check and correct the assignment.
2. Have students read and study the examples on page 110 to determine the method being used to find the greatest common factor. Work an example at the board. Let the students tell you what to do.
3. Use another example or two asking students to use this method. (Example: Find the greatest common factor of 280 and 315.)

Summary:

Discuss the oral, page 110.

Suggested Assignment:

Work the written, page 110.

Lesson 7

Relatively Prime Numbers

Aim:

To teach the student to use the greatest common factor in reducing fractions to lowest terms.

Suggested Methods:

Discovery, discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Use the first example on page 111 to help define the term, "relatively prime."
3. Discover that a fraction is in lowest terms when the numerator and denominator are relatively prime. Use a fraction to reduce such as $\frac{8}{20}$. Ask the students how they know that $\frac{2}{5}$ is in lowest terms.
4. Have students identify the greatest common factor in the examples on page 111.
5. Use the example $\frac{156}{420}$ and show how the greatest common factor can be divided out. $\frac{156}{420} = \frac{2^2 \times 3 \times 13}{2^2 \times 3 \times 5 \times 7} = \frac{13}{5 \times 7} = \frac{13}{35}$

   $2^2 \times 3$ was the greatest common factor.

Summary:

Use the oral, page 111.

Suggested Assignment:

Work the remainder of the written, page 111.

Lesson 8

Least Common Multiple

Aim:

To teach the students to find the least common multiple of a set of numbers.

Suggested Methods:

Definition, discussion, and demonstration.

Suggested Materials:

Transparencies, overhead projector, colored pens

Developmental Steps:

1. Check and correct the assignment.
2. Define the term "least common multiple."
3. Discuss the example on page 112, listing multiples of 2 and 3, on transparency or the board.
4. Identify the common multiples. A different colored pen could be used to check these.
5. Identify the least common multiple. A third color might be used to check it.
6. Ask when we might need to know the least common multiple of a set of numbers. (Finding the lowest common denominator)
7. Discuss the steps followed in finding the least common multiple in the following example: Find the least common multiple of 21, 35, 56, and 6.
8. Discuss examples like those on page 113.

Summary:

1. Review the definition of least common multiple.
2. Discuss the oral exercises, page 113.

Suggested Assignment:

Work the written exercises, page 113.

Lesson 9

Factors of Integers

Aim:

To extend the students' understanding of factors to include intergers.

Suggested Methods:

Review and discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Using the assignment, show that once terms are factored, it is easy to find the greatest common factor.
3. Discuss the examples in the first column, page 114, to demonstrate the factorization of negative integers. (Review the rules for the multiplication of integers.)
4. Discuss the meaning of $(-3)^2$ to show that it is positive and not negative. Also discuss the meaning of -3^2 to show that it is negative. Show that care must be exercised in using powers of negative numbers.
5. Emphasize that there are several acceptable factorizations of a negative integer as with -42 on page 114.

Summary:

Discuss the oral exercises, page 114.

Suggested Assignment:

Work the written exercises, page 114, and possibly some of the problems on page 115, as 1, 2, 4, 5, and 9.

Lesson 10

Chapter Review

Aim:

To review the work covered in the chapter.

Suggested Methods:

Review

Developmental Steps:

1. Check, discuss, and correct the assignment.
2. Using pages 117-118 as a guide, review the work covered in the chapter. Work as many sample problems as time allows and student needs indicate.

Suggested Assignment:

Study for the chapter test.

Lesson 11

Chapter Test

Aim:

To test the students' behavior as stated in the behavioral objectives for Unit VI.

Suggested Materials:

Mimeographed tests

Developmental Steps:

Use the chapter test on page T 118 as it is, or use parts 1, 2, and 5 and combine some of the parts 3 and 4, asking the students to find both the greatest common factor and the least common multiple of each set.

Unit VII

Rational Numbers

Unit VII

Rational Numbers

Behavioral Objectives:

1. The student will define the set of rational numbers.
2. The student will determine that the properties of the operations and of one and zero hold for the set of rational numbers.
3. Given rational numbers, the student will correctly add, subtract, multiply, or divide giving the answer in simplest form.
4. Given fractions, the student will change those fractions to repeating or terminating decimals.
5. Given repeating or terminating decimals, the student will change them to fractions.

Lesson 1

The Set of Rational Numbers

Aim:

To introduce the student to the set of rational numbers.

Suggested Methods:

Review and discussion.

Suggested Materials:

Transparency number 511 - (306)

Developmental Steps:

1. Return, discuss and have the students correct their errors on the chapter test.
2. Draw a number line on the board and use it to demonstrate points which correspond to numbers as you discuss together the material on page 119, including the oral exercises.
3. Show and discuss the transparency.
4. Using the example equation on page 120, review the meaning of the replacement set.
5. Define the set of rational numbers. Emphasize that all integers are rational numbers and that the set of integers is a subset of the set of rational numbers. Give some examples to demonstrate this.

Summary:

Discuss the exercises, page 120.

Suggested Assignment:

Work the written exercises, page 120.

Lesson 2

Other Names for Rational Numbers

Aim:

To review with the students the methods of changing mixed numbers to fractions and vice-versa.

Suggested Methods:

Review, discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material on page 121. Review the definition of rational number. Name several examples, such as $\frac{1}{8}$, $\frac{11}{4}$, $\frac{9}{1}$, etc.
3. Put $\frac{13}{5}$ on the board and ask, "Why do we divide 13 by 5 to change this to a numeral greater than one?" (Because there are 5 fifths in each whole and we want to convert as many fifths to wholes as we can)
4. Put 3 2/7 on the board and ask, "Why do we multiply 7 times 3 to change this to rational number form?" (Because there are 7/7 in each whole and we want to convert these wholes to 21/7 then add the 2/7 to it)
5. Discuss the other examples.

Summary:

Use the oral exercises, page 121.

Suggested Assignment:

Work the written exercises, page 121.

Lesson 3

Equivalent Fractions

Aim:

To assist the student in reviewing the methods of writing equivalent fractions.

Suggested Methods:

Review, definition, and discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Review the division property of equations.
3. Discuss the examples on page 122 and 123. Let the students justify each step in the examples.
4. Define the term equivalent and review what is meant by "simplest name" for a fraction.
5. Work together numbers 1, 6a, 8c, and 9a from the written exercises.

Summary:

Do the oral exercises, page 123.

Suggested Assignment:

Do the remainder of the written exercises, page 123.

Lesson 4

Multiplication of Rational Numbers

Aim:

1. To teach the student to multiply rational numbers.
2. To review the properties of multiplication with respect to rational numbers.

Suggested Methods:

Review, discussion, demonstration.

Suggested Materials:

Flannel board and fraction cutouts

Developmental Steps:

1. Check and correct the assignment.
2. Use the flannel board and cutouts to show the examples in the first three paragraphs.
3. Develop and prove rule $\frac{a}{n} \times \frac{c}{r} = \frac{ac}{nr}$. Since $\frac{ac}{nr}$ is a rational number, the closure property holds for rational numbers. Develop the associative property and the commutative property of rational numbers. Show that 1 is also the identity element for rational numbers. Discuss these fully.
4. Work together examples 1a, 5b, 6a, 9a.

Summary:

Discuss the oral exercises, page 125.

Suggested Assignment:

Do the remainder of the written exercises, page 125.

Lesson 5

Multiplication (Continued)

Aim:

To strengthen the students' concepts of multiplication of rational numbers.

Suggested Methods:

Review, discussion, and practice.

Developmental Steps:

1. Check and correct the assignment.
2. Review prime factors.
3. Discuss the examples on page 126. Have the students justify each step in the examples.
4. Use some examples from page 341 for practice.

Summary:

Discuss the oral exercises, page 126. Work the example by multiplying and then reducing and by factoring.

Suggested Assignment:

Work the remainder of the written exercises.

Lesson 6

Division of Rational Numbers

Aim:

1. To teach the students how to determine the reciprocal of any non-zero rational number.
2. To teach the students to divide rational numbers.

Suggested Methods:

Discussion, investigation, demonstration, practice.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the example, page 127.
3. Define the term reciprocal and demonstrate how it is found by using positive and negative fractions.
4. Work together examples 1a, 2c, and 5c on page 127.
5. Discuss the examples of division on pages 128 and 129.
6. Introduce to the students the method of dividing fractions by using the least common denominator. $9/16 \div 3/8 = 9/12 \div 6/16 = \frac{9 \div 6}{16 \div 16} = \frac{1\frac{3}{6}}{1} = 1\frac{3}{6} = 1\frac{1}{2}$
7. Work together examples 1a, 3b, and 5a on page 129.

Summary:

Discuss the oral exercise on pages 127 and 129.

Suggested Assignment:

Do the remainder of the written exercises on page 127 and problems 1-10 in the written exercises on page 129.

Lesson 7

Division (Continued)

Aim:

Same as the the previous lesson.

Suggested Materials:

Mimeographed sheet on the division of fractions

Suggested Methods:

Review, practice.

Developmental Steps:

1. Check and correct the assignment.
2. Work together problems 11-16 on page 129.
3. Have students practice with problems on mimeographed sheets. These may be some you make up or get from other texts or workbooks.

Summary:

Discuss oral exercise on page 129.

Suggested Assignment:

Do problems 1-10 on dividing rational numbers, page 341.

Lesson 8

Addition of Rational Numbers

Aim:

1. To have the students review the least common multiple as a method of finding the lowest common denominator.
2. To review the addition of rational numbers.

Suggested Methods:

Discussion, review, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Review the addition of like fractions.
3. Write on the board an example of unlike fractions, as $1/3 + 1/2$. Ask, "Why can't we add them?" (Because they are parts of different sizes)
4. Review the least common multiple as a method of finding the lowest common denominator.
5. Use examples to examine the addition of rational numbers with respect to the various properties as discussed on pages 130-131. Discuss and develop the properties.
6. Work together examples 5c and 9d.

Summary:

Discuss the oral exercises, page 131.

Suggested Assignment:

Work the remainder of the written exercises, page 131.

Lesson 9

Addition (Continued)

Aim:

To assist the students in extending the addition of rational numbers to include mixed numbers.

Suggested Methods:

Review, discussion, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the problem illustrated in the first column, page 132. Go through the steps of the example, letting students tell "why" in each case.
3. Demonstrate the vertical form at the bottom of the page.
4. Have them work $\begin{array}{r} 11\ 5/6 \\ +\ 19\ 3/7 \end{array}$ and $\begin{array}{r} 37\ 13/21 \\ +\ 42\ 15/24 \end{array}$

   Walk around to check the form and the accuracy of each student's work, giving individual help where needed.
5. Emphasize that finding the least common multiple is the quickest way to find the denominator.

Summary:

Discuss the oral questions on page 132.

Suggested Assignment:

Work the written exercises, page 132.

Lesson 10

Subtraction of Rational Numbers

Aim:

To review with the students the subtraction of rational numbers.

Suggested Methods:

Review, discussion, demonstration, and practice.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material on page 133 on negative rationals.
3. Answer the oral questions as a class activity.
4. Have students answer the written examples orally.
5. Review the rules for addition of integers as: $7 - 3 = 7 + (-3)$ and $5 - 8 = 5 + (-8)$.
6. Review the subtraction of like fractions.
7. Discuss the examples on page 134.
8. Work together examples 4a and 8a.

Summary:

Discuss the oral exercises, page 134.

Suggested Assignment:

Work the remainder of the written exercises, page 134.

Lesson 11

Distributive Property

Aim:

1. To illustrate to the students the distributive property with respect to the set of rational numbers.
2. To have the students practice the operations with rational numbers.

Suggested Methods:

Investigation, discovery, discussion, and demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Divide the class roughly in half and ask one half to work 1a in the written exercises and the other to work 1b, page 135.
3. When corrections are made, ask why both sections got the same answer. (Illustrates the distributive property)
4. Reverse the sections for examples 2a and 2b.
5. Remind them that these are mixed operations and, therefore, we must perform the operations in parentheses first.
6. Have all the class work 3a and 3b as practice.

Summary:

1. Discuss the oral exercise, page 135.
2. Review briefly methods of addition, subtraction, multiplication, and division of rational numbers.

Suggested Assignment:

1. Work the written exercises 8-10, page 135. (For more advanced classes include 6 and 7.)
2. Do the quick review, page 135.

Lesson 12

Problem Solving

Aim:

To strengthen the students' problem solving abilities.

Suggested Methods:

Practice, discussion.

Developmental Steps:

1. Check and correct the assignment. Have students put some of the examples on the board and explain them.
2. Read together, discuss, and work together problems 5, 7, and 10, page 136.

Summary:

Discuss the steps used to solve a problem.

Suggested Assignment:

1. Work the remainder of the problems on page 136 except number 11.
2. Do page 164, part 2, column b, 1-6. (Number 11 page 136 may be optional for those who want to try it.)
3. There will be a test tomorrow on mixed operations.

Lesson 13

Test

Aim:

To test the students on operations with rational numbers.

Suggested Methods:

Testing

Developmental Steps:

1. Check and correct the assignment.
2. Give a test including one step problems in addition, subtraction, multiplication, and division; and some with mixed operations as in the quick review on page 135.

Suggested Assignment:

None

Lesson 14

Decimal Numerals

Aim:

To extend the students' knowledge of the set of rational numbers to include decimal numerals.

Suggested Methods:

Discussion, review, demonstration.

Developmental Steps:

1. Return the tests: discuss and correct the problems missed.
2. Draw a place value chart on the board. Shade the units place to show it as the middle. Place the decimal and show it divides the whole numbers from the fractions.
3. Show the balance between the 10^1 and $1/10^1$ places, 10^2 and $1/10^2$ places, etc.
4. Show that the numerator of a decimal fraction is shown by the numerals written and the denominator is indicated by the number of places to the right of the decimal.
5. Emphasize that all integers are decimal numerals.
6. Review the proper way to read decimal numerals.
7. Use some examples to show how to write fractions as decimals and and decimals as fractions. (as 1a, 1b, and 4a, 4b, page 137)

Summary:

Discuss the oral exercises, page 137.

Suggested Assignment:

Work the remainder of the written exercises, page 137.

Lesson 15

Addition and Subtraction Using Decimals

Aim:

To review with the students the addition and subtraction of decimal numerals.

Suggested Methods:

Discussion, demonstration, and review.

Developmental Steps:

1. Check and correct the assignment.
2. Read aloud the first two paragraphs on page 138. Ask, "How do we find a common denominator for decimal fractions?"
3. Read together the remainder of the material on page 138. Use examples to show that calculations with decimal numerals give the same result as calculations with corresponding fractional numbers.

Summary:

Work together the examples in the oral exercises, page 138.

Suggested Assignment:

Work the written exercises and problems, page 139.

Lesson 16

Multiplication and Division Using Decimals

Aim:

To have the students review multiplication and division with decimal numerals.

Suggested Methods:

Discussion, demonstration, and review.

Developmental Steps:

1. Check and correct the assignment.
2. Put the example $\begin{array}{r} .13 \\ \times \quad .3 \end{array}$ on the board.
3. Work the example and ask the students how to place the decimal (.039).
4. Ask, "Why does this method of counting the decimal places work?" (Becuase 3/10 x 13/100 = 39/1000)
5. Work examples 1a, 1b, and 1c in the written exercises with the assistance of the students.
6. Put the example 2.07 ÷ .23 on the board. Illustrate the use of the caret as $.23\overline{)2.07}$. Emphasize that we are really multiplying both numbers by 100. (as $\frac{2.07 \times 100}{.23 \times 100} = \frac{207}{23}$)
7. Work together examples 4a and 4b in the written exercises.

Summary:

Review the methods of placing decimals in multiplication and division of decimal numerals.

Suggested Assignment:

Work the remainder of the written exercises, page 141.

Lesson 17

Terminating and Repeating Decimals

Aim.

1. To teach the student to identify some fractions that will produce terminating decimals.
2. To review with the students the changing of fractions to decimal numerals.

Suggested Methods:

Discussion, demonstration, practice.

Developmental Steps.

1. Check and correct the assignment.
2. Discuss the material in the first column, page 142. Use one or two examples (7/40) on the board to illustrate pairing two's and five's to make the denominator of a fraction a power of ten so the fraction can be changed to a decimal. Illustrate each example by using both multiplication and division.
3. Answer the oral exercises, page 142, as a student activity.
4. Define terminating and repeating decimals.
5. Demonstrate, with one or two examples, the proper way to write a repeating decimal. ($.6\overline{363}...$)
6. Discuss and answer the oral exercises, page 143.

Summary:

1. Review how to tell whether a decimal will terminate or repeat.
2. Define terminating and repeating decimals. Emphasize that every rational number can be named by either a terminating or a repeating decimal.

Suggested Assignment:

Work the written exercises, page 142, numbers 3 and 4, and the written exercises, page 143.

Lesson 18

Changing Decimals to Fractions

Aim:

1. To review with the students how to change terminating decimals to fractions.
2. To teach them a method of changing repeating decimals to fractions.

Suggested Methods:

Review, discussion, demonstration.

Suggested Materials:

Transparency, pens, and overhead projector

Developmental Steps:

1. Review with students the changing of terminating decimals to fractions. (as on page 137)
2. Select an example of a repeating decimal such as $.5\overline{151}...$ Demonstrate how to change it to a fraction. Use the overhead projector for this.
3. Select another example such as 1a, page 144, and let a student (working at the overhead projector) change it to a fraction. Allow other students to assist if he has trouble changing it.

Summary:

Work together the examples in the oral exercises.

Suggested Assignment:

Work the written exercises 1-3. (It is suggested that the examples in 4, 5, and 6 are too difficult for most classes.) If time allows in this period, page 145 can be discussed.

Lesson 19

Chapter Review

Aim:

To review the chapter in preparation for a chapter test.

Suggested Methods:

Review, practice.

Developmental Steps:

1. Check and correct the assignment.
2. Using pages 147 and 148 as a guide, review the chapter. Work as many examples as time permits.

Suggested Assignment:

Chapter test tomorrow

Lesson 20

Chapter Test

Aim:

To test the students' behavior as stated in the behavioral objectives for Unit VII.

Suggested Methods:

Testing

Suggested Materials:

Mimeographed test

Developmental Steps:

Give a test which might include:

1. Part 1 of the "Independent Test," page T 148.
2. A section very similar to part 2, page 148.
3. Fractions to change to terminating and repeating decimals.
4. Terminating and a few simple repeating decimals to change to fractions.
5. "The set of rational numbers" to be defined.

Suggested Assignment:

None

Unit VIII

Ratio, Proportion, and Per Cent

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Ratio, Proportion, and Per Cent

Behavioral Objectives:

1. The student will demonstrate his understanding of ratio as the relation between the numbers of two sets which is expressed as an ordered pair of numbers, by using ratios to solve problems.
2. Given ratios and rates, he will express them in their simplest form.
3. Given problems, the student will form proportions and correctly solve them.
4. He will use per cent as a special kind of ratio in solving problems.

Lesson 1

Ratio

Aim:

To teach the ordered pair concept of ratio, and to express ratios in their simplest form.

Suggested Methods:

Review, observation, discussion.

Developmental Steps:

1. Return and correct the tests.
2. Explain that the next lessons will be from chapter 12, as this material is very important. Discuss some of the uses of ratio and per cent.
3. Look together at the illustration at the top of page 231. Discuss the ratio of each example as sets of stars to sets of boxes to sets of stars. Emphasize that the order is important.
4. Review equivalent fractions showing that the same ratio may be expressed in different ways.
5. Read, answer, and discuss the oral questions, page 231.
6. Answer together the oral exercises, page 232.

Summary:

Define ratio again. Discuss what is meant by "in simplest form".

Suggested Assignment:

Work the written exercises, page 232.

Lesson 2

Ratio and Measurements

Aim:

To help the students understand the use of measures in a ratio and to interpret a rate as a special kind of ratio.

Suggested Methods:

Discussion, demonstration, practice.

Developmental Steps:

1. Check, correct, and discuss the assignment.
2. Set up 8:20 and 12:30 as ratios. Show they name the same ratio in simplest fractional form.
3. Give examples, one at a time, of ratios involving measures and rates. Let the students determine each time whether the measures should be expressed as a ratio or as a rate. Then ask them to express each in simplest form.

Summary:

Discuss the oral questions, page 233.

Suggested Assignment:

Work the written exercises, page 233, 1-8. Number 9 might be optional for most average classes, or included for a more advanced class.

Lesson 3

Proportion

Aim:

To help the students understand the meaning of a proportion and to identify the means and extremes.

Suggested Methods:

Definition, discussion, investigation.

Suggested Materials:

Transparency showing a proportion with means and extremes labeled.

Developmental Steps:

1. Check and correct the assignment.
2. Set up two ratios such as 8:20 and 12:30. Reduce them to simplest form to show they are names for the same ratio. Set them up to show they are equal as: $\frac{8}{20} = \frac{12}{30}$. Point out that this statement of equality of two ratios is called a proportion.
3. Define: proportion, means, extremes. Point these out on the transparency.
4. Read together the first paragraph, second column. Answer questions asked in the paragraph.
5. Use the examples in the remainder of the second column to show that fractions (ratios) having the same denominator are equal if, and only if, the numerators are equal.

Summary:

Read together and discuss the oral questions, page 235.

Suggested Assignment:

Work the written exercises, page 235. Individual students may be assigned exercises 1, 2, and 3 in "Something to do," page 235, and asked to report to the class on them.

Lesson 4

Solving Proportions

Aim:

To teach the student to solve proportions and to solve verbal problems by using proportions.

Suggested Methods:

Discussion, discovery, demonstration.

Developmental Steps:

1. Check and correct the assignment (1-7 in the written exercises, page 235).
2. Have students report on the assigned exercises, page 235. Lead them to discover (a) that the product of the means is equal to the product of the extremes; (b) that enterchanging the means or extremes or both produces another proportion; (c) that the sum of the first and third terms is to the sum of the second and fourth term as the first is to the second or as the third is to the fourth. Use many examples to set up patterns so the students may discover the above.
3. Emphasize that a proportion is an equation and can be solved as one. Use examples on page 236 to illustrate this.
4. Discuss the example problem, page 237, to demonstrate how verbal problems can be solved by using proportions.
5. Work together problems 1 and 2, page 237.

Summary:

Solve together some of the proportions in the oral, page 236.

Suggested Assignment:

Work problems 4-6 in the written exercises, page 236, and problems 3-9, page 237.

Lesson 5

More About Rates

Aim:

To teach the students that every rate has many names, depending on the units of measure used.

Suggested Methods:

Question and answer, review, and discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Read together and discuss column one on page 238. Discuss other methods of solving these two problems. Compare the methods.
3. Review the tables of measure, page 376. Refer the students to them for any needed measures.

Summary:

Read together, discuss, and work the oral questions, page 238.

Suggested Assignment:

Work the written exercises, 1-5, page 238.

Lesson 6

Per Cent

Aim:

To introduce to the students the idea of per cent as a special kind of ratio, and to teach them to express a ratio as a per cent and a per cent as a ratio.

Suggested Methods:

Discussion, review, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Read together and discuss the material in column one, page 239.
3. Discuss the example at the top of column two. Review equivalent fractions (from page 123). Stress the meaning of per cent as per hundred.
4. Read together and answer numbers 1 and 2 in the oral exercises.
5. Read number 3 in the oral exercises. Devise a rule for changing a decimal numeral to a per cent. (Move the decimal two places to the right and add the per cent sign.)
6. Read number 4 in the oral exercises. Devise a rule for changing a per cent to a decimal numeral. (Move the decimal two places to the left and drop the per cent sign.) Review changing decimal numerals to fractions to answer the question.

Summary:

Work together examples 1a, 2c, and 4a in the written exercises.

Suggested Assignment:

Work the remainder of the written exercises, page 239.

Lesson 7

Per Cent (Continued)

Aim:

To teach the students to solve per cent problems by using proportion, and to introduce the per cent formula.

Suggested Methods:

Review, discussion, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss and emphasize that a per cent cannot be used as a per cent to solve problems. Discuss the different forms of some per cents as: $20\% = .20 = \frac{20}{100} = 1/5$
3. Read together and discuss the example problem in the first column, page 240. Use proportion to solve it.
4. Introduce the terms: base, rate, and percentage (or part). Emphasize that the base (the whole) is always equal to 100%, that the rate is the per cent, and that the percentage is sually part of the whole, but may be more than the base.
5. Identify base, rate, and percentage in the previous example.
6. Read together and discuss the next example, identifying the parts.
7. Introduce the per cent formula.
8. Use the three examples on page 241 to demonstrate that the per cent formula can be used to find the number that is either the rate, the base, or the percentage, providing the other two numbers are known. Review the properties of equations as you work the examples.

Summary:

1. Discuss number one in the oral questions.
2. Have the students use the per cent formula to solve the three examples in number 2.

Suggested Assignment:

Work the written exercises, page 241.

Lesson 8

Equivalent Fractions and Per Cents

Aim:

To teach the student when and how he should use the fraction equivalent of a per cent to solve a problem.

Suggested Methods:

Discussion, review, and demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Use number 3 to illustrate the different ways of solving a per cent problem. (proportion and formula)
3. Demonstrate solving the same problem using the fraction form 2/5 for 40%.
4. Read together and discuss the first example on page 242. Emphasize that the decimal form of 16 2/3% does not yield an exact answer.
5. Review methods of changing per cents to decimals and fractions and fractions to decimals and per cents.
6. Discuss together the oral exercises and the table at the top of page 242.

Summary:

Review the methods of solving per cent problems.

Suggested Assignment:

Solve each problem in the written exercises two ways. (page 242)

Lesson 9

Solving Problems

Aim:

To teach the student to apply the general percentage formula to the solution of problems involving discount, tax, and commission.

Suggested Methods:

Discussion and demonstration.

Suggested Materials:

Newspaper advertisements showing ways of expressing discounts

Developmental Steps:

1. Correct the assignment by having students put the different forms of each problem on the board. Discuss each one.
2. Discuss buying an article at a discount. Show the newspaper advertisements containing such phrases as: 1/3 off, reduced 10%, or 20% off, etc. Let the students tell you how to find the discounts on some of the articles advertised. Then state the rule: discount = regular price x rate.
3. Discuss the meaning of tax and how to find the amount of tax on an article being purchased. Discuss commission and how to calculate the amount received as a commission.
4. Identify base, rate, and percentage in each example.

Summary:

Read together and discuss the problems (1-8) page 243. Identify base, rate, and percentage in each. (orally)

Suggested Assignment:

Solve problems 1-8, page 243.

Lesson 10

Chapter Review

Aim:

To review chapter 12.

Suggested Methods:

Review, discussion, and drill.

Developmental Steps:

1. Discuss and correct the assignment.
2. Using pages 245-246 as a guide, review the material covered in the chapter.
3. Work together examples (as many as time permits) from page 246.

Suggested Assignment:

Chapter test tomorrow

Lesson 11

Test

Aim:

To test the students' ability to use ratio and proportion to solve problems and to solve per cent problems of different kinds.

Suggested Methods:

Testing

Developmental Steps:

The Independent Test for Chapter 12, page T 246 may be used as it is.

Suggested Assignment:

None

Unit VIII

Ratio, Proportion, and Per Cent

An Alternate Plan for Teaching Per Cent

Pages 239-243

1st Day.

Introduction

Use page 239, 1st column. Discuss the meaning of per cent and its uses. Emphasize that per cent means hundredths.

1. Use the oral questions and discussion to evolve the following rules: To change a percent to a decimal, move the decimal two places to the left and drop the per cent sign. To change a decimal to a per cent move the decimal two places to the right and add the per cent sign.
2. Give mimeographed sheet for practice and assignment for next day.

Assignment for the Second Day

	Change to per cents		Change to decimals	
	(Remember: Per cent is hundredths. 1 whole is $\frac{100}{100}$ or 100%, $\frac{1}{100}$ = 1%, etc.)			
I.	(1) .16		(1) 38%	
	(2) .75		(2) 67%	
	(3) .08		(3) 3%	
	(4) .02		(4) 9%	
	(5) .05	I. (1-15 are between 1%	(5) 40%	
	(6) .80	and 100%)	(6) 90%	
	(7) .39		(7) 16%	
	(8) .6		(8) 38%	
	(9) .158		(9) 43.8%	
	(10) .635		(10) 56.1%	
	(11) .054		(11) 12%	
	(12) .06		(12) 5%	
	(13) .039		(13) 10.2%	
	(14) .46		(14) 2%	
	(15) .046		(15) 12.55%	
II.	(1) .002		(1) .01%	(7) 3/4%
	(2) .008		(2) .25%	(8) 1/2%
	(3) .001	II. Less than 1%	(3) .02%	
	(4) .0015		(4) .36%	
	(5) .0023		(5) .1%	
	(6) .0075		(6) .5%	
III.	(1) 2.25		(1) 300%	
	(2) 5.75		(2) 250%	
	(3) 2	III. More than 100%	(3) 175%	
	(4) 4		(4) 1000%	
	(5) 1.5		(5) 640%	
	(6) 10		(6) 108%	

2nd Day:

1. Check the assigned mimeographed sheet.
2. Give the attached check test.
3. Discuss changing a per cent to a decimal and then to its fractional form.

   Examples:

Per Cent	Decimal	Fraction
35%	.35	35/100 = 7/20
5%	.05	5/100 = 1/20
60.5%	.605	605/100 = 121/200
.2%	.002	2/1000 = 1/500
175%	1.75	1 75/100 = 1 3/4
3/4%	.0075	75/10000 = 3/400

4. Give mimeographed sheet assignment for 3rd day. (Attached)

Test for the Second Day

1. Write the rule for changing a decimal to a per cent.

   Change to per cents:

2. .07

3. .39

4. .456

5. .0057

6. 1.52

7. Write the rule for changing a per cent to a decimal.

   Change to decimals.

8. 2%

9. 18%

10. 16.5%

11. .05%

12. 109%

Assignment for the 3rd day to be handed out the 2nd day

Percent	Decimal	Fraction
1. 50%		
2. 25%		
3. 75%		
4. 40%		
5. 80%		
6. 30%		
7. 35%		
8. 48%		
9. 16%		
10. 6%		
11. 8%		
12. .5%		
13. .2%		
14. .06%		
15. 12.5%		
16. 7.5%		
17. 11.60%		
18. 62.5%		
19. 150%		
20. 300%		

3rd Day:

1. Return and correct the tests.
2. Check the practice sheets assigned.
3. Give the test. (attached)
4. Discuss changing a fraction to a decimal to a per cent.

   Examples:

Fraction	Decimal	Per Cent
1/5	.2	20%
17/20	.85	85%
3/8	.375	37.5%
1/200	.005	.5%
3 7/8	3.875	387.5%

5. Assign practice. (attached)

Test, 3rd Day

	PERCENT	DECIMAL	FRACTION
1.	50%		
2.	9%		
3.	18.75%		
4.	4.5%		
5.	600%		
6.	60%		
7.	6%		
8.	.6%		

Assignment for the 4th day to be handed out the 3rd day

	FRACTION	DECIMAL	PER CENT
1.	1/2		
2.	1/4		
3.	3/4		
4.	1/5		
5.	3/5		
6.	1/8		
7.	7/8		
8.	1/10		
9.	3/10		
10.	9/10		
11.	1/16		
12.	3/20		
13.	17/50		
14.	3/50		
15.	1/40		
16.	3/1000		
17.	1/125		
18.	2 1/4		
19.	17 7/20		
20.	6		
21.	81/80		
22.	13		

4th Day:

1. Return and correct the tests.
2. Check and correct the assignment.
3. Discuss the following mixed practice.

   Examples:

FRACTION	DECIMAL	PER CENT
_____	.24	_____
3/8	_____	_____
_____	_____	.06%
_____	.054	_____

4. Assign mixed practice. (attached)

Assignment for 5th day to be handed out the 4th day

	FRACTION	DECIMAL	PER CENT
1.	1/8	_____	_____
2.	_____	.27	_____
3.	_____	_____	3%
4.	_____	.07	_____
5.	2/3	_____	_____
6.	_____	_____	150%
7.	9/20	_____	_____
8.	_____	_____	.3%
9.	_____	.062	_____
10.	_____	.005	_____
11.	2 4/5	_____	_____
12.	_____	_____	12.5%
13.	_____	.75	_____
14.	_____	2.25	_____
15.	_____	.0015	_____
16.	7/10	_____	_____
17.	3 7/16	_____	_____
18.	_____	50	_____
19.	10/2	_____	_____
20.	_____	_____	50%
21.	_____	.05	_____
22.	1/200	_____	_____
23.	_____	_____	.05%

5th Day:

1. Check and correct the assignment.
2. Test on mixed practice.

TEST 5th DAY

	FRACTION	DECIMAL	PER CENT
1.	1/5	_____	_____
2.	_____	.006	_____
3.	_____	5.8	_____
4.	7/500	_____	_____
5.	_____	.85	_____
6.	_____	_____	360%
7.	1 2/5	_____	_____
8.	_____	_____	7%
9.	_____	_____	90%
10.	_____	3.33	_____

3. Assign practice in multiplication and division with decimals.

6th Day:

1. Return and correct the tests.
2. Check the assignment.
3. Discuss pages 240-241.
4. Discuss oral number 1 together. Work together the problems in oral number 2. (Use the form illustrated on the mimeographed sheet.)
5. Pass out the mimeographed sheet for practice and assignment. (attached)

Assignment for the 7th day to be handed out the 6th day

$p = rb$

p = percentage - part
r = rate or per cent
b = base, the whole (=100%)

I. Find part

1. Find 26% of 30. 7.8
2. 3% of 67 is 2.01.
3. .5% of 120 is .60.

(examples)

25% of 48 is _____.

$p = rb$
$p = .25 \times 48$
$p = 12$

II. Find base

1. $1.44 is 6% of $24.
2. 30% of 500 is 150.
3. 330 is 150% of 220.

12 is 25% of _____.

$p = rb$
$\frac{12}{25} = \frac{.25 \times b}{.25}$
$48 = b$

III. Find rate

1. 24 is 48 % of 50.
2. 20 % of 77 is 15.40.
3. 7 is .5 % of 1400.

12 is _____% of 48.

$p = rb$
$\frac{12}{48} = \frac{r \times 48}{48}$
$.25 = r$
$25\% = r$

7th Day:

1. Check and correct the assignment. (Recognize base, rate and part (percentage) in each.)
2. Discuss equivalent fractions, page 242. Show examples of working problems with decimal and fractional forms of per cent.
3. Assign page 241, written problems 1-6 and page 242, written problems 1-6. (Require the use of fraction form on page 242.)

8th Day:

1. Check and correct the assignment. Emphasize that a repeating decimal will not give an exact answer.
2. Pass out mimeographed sheets for more practice. (attached) Insist on correct equation form.

Assignment for 9th day to be handed out the 8th day

1. 47% of 850 is 399.50.
2. 87 1/2% of 848 is 742.
3. 24 is 48% of 50.
4. 30% of 500 is 150.
5. 180 is 6% of 3000.
6. 8 is 12.5% of 64.
7. 25% of 48 is 12.
8. 13 is 25% of 52.
9. 75% of 60 is 45.
10. 33 1/3% of 681 is 227.
11. 60% of 400 is 240.
12. 3 is 30% of 10.
13. 8 is 40% of 20.
14. 44 is 80% of 55.
15. 17 is 85% of 20.
16. 40% of 200 is 80.
17. 125% of 296 is 370.
18. 117% of 105 is 122.85.
19. 140 is 175% of 80.
20. 300% of 8 is 24.
21. 15 is 18.75% of 80.
22. 2% of 500 is 10.
23. 5.4 is 6% of 90.
24. 5% of 500 is 25.

9th Day:

1. Check and correct the assignment.
2. Test (attached). It is suggested that some points be counted on the test for correct form.

Test, 9th Day

Solve:

1. 31% of 75.6 is 23.4.
2. 85 is 50% of 170.
3. 28% of 778 is 217.84.
4. 146.25 is 65% of 225.
5. .5% of 171 is .855.
6. 200% of 300 is 600.
7. 13 is 25% of 52.
8. 8 is 12.5 % of 64.
9. 100% of 270 is 270.
10. 66 is 15% of 440.
11. 91 is 16 2/3% of 346.
12. 66 2/3% of 15 is 10.

10th Day:

1. Return and correct the tests.
2. Discuss translating problems into per cent statements.

   Example: Number 7 page 241

   18 is _____% of 24.

3. Assignment: Page 241, problems 7-12

   Translate and then work.

11th Day:

1. Check and correct the assignment.
2. Discuss (page 243) discount, commission and tax as per cent.
3. Read some of the problems and discuss them.
4. Assignment, page 243. Work the written problems 1-8.

12th and 13th Day:

Continue with chapter review and chapter test as in the other plan.

Unit IX

Points, Lines, and Planes

Unit IX

Points, Lines, and Planes

Behavioral Objectives:

1. The student will demonstrate an understanding of the terms: point, line, line segment, ray, polygon, parallel, perpendicular, angle, and plane, by identifying examples, defining the terms, or completing statements about them.
2. Given lines and/or planes, the student will state the intersections of those given lines and/or planes.
3. Given polygons, he will properly name the polygons, their sides and diagonals.
4. Given intersecting lines, the student will identify adjacent and vertical angles.
5. The student will measure any given angle using a protractor.
6. He will use the protractor to draw a model of an angle of any given measure in degrees.

Lesson 1

Points and Lines and Subsets of a Line

Aim:

To help the students develop concepts of points and lines that are consistent with the requirement of the deductive system of mathematics called geometry. To show the students that certain subsets of a line can be formed and precisely identified.

Suggested Methods:

Discussion, drill.

Developmental Steps:

1. Return the tests; check and discuss them.
2. Discuss the material on page 149. Emphasize that a string, the edge of a page, etc., are really only models of lines, as a line has length but no thickness and no width. It is an idea.
3. Define point and line. Stress that we use models of these when we place a dot on a sheet of paper, etc.
4. Review subsets.
5. Name and define the subsets of a line. Using a model, represent a line, a line segment, and the endpoints of the segment. Represent a ray, letter its vertex, and discuss how a ray is named.

Summary:

Discuss the oral exercises on pages 149 and 150.

Suggested Assignment:

Study the material that is to be used with pages 149 and 150 on page 344. Draw the models that you are instructed to represent on that same page.

Lesson 2

Intersecting Lines and Planes

Aim:

To develop an understanding of intersecting lines. To discover some of the basic relationships between a line and a plane and between two planes.

Suggested Methods:

Discussion, demonstration.

Suggested Materials:

Transparency with a diagram of two intersecting lines, one with several intersecting lines and some blank acetate sheets.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material on page 151. Use a transparency that has a diagram of intersecting lines to discuss the material. On a blank acetate sheet, put one point (model). Use it to demonstrate the numbers of lines that can be determined by one point. Repeat the process to demonstrate the number of lines determined by two points. Discuss intersection. Develop the idea that two lines can intersect in only one point.
3. Do over 1a and 4b together in class.
4. Discuss the material on pages 152 153. Develop the idea of a plane and the conclusions about the plane.
5. Do numbers 1 and 7 in the written exercises, page 153.

Summary:

Discuss the oral exercises on pages 151 and 153.

Suggested Assignment:

Work the remainder of the written exercises on pages 151 and 153.

Lesson 3

Parallel Lines and Planes

Aim:

To develop within the students an understanding of parallelism regarding two lines, a line and a plane, and two planes.

Suggested Methods:

Discussion, question and answer.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material on pages 154-155. Discuss parallelism of lines, planes, a line and a plane. Let the students suggest the possibilities that may exist if two lines are in different planes. Develop the idea of the possibilities that may result when two lines are drawn in a plane.
3. Define parallel lines, skew lines, parallel planes, and intersecting lines.
4. Read, discuss and answer numbers 1 and 10 under the written exercises on page 155.

Summary:

Discuss the oral exercise on page 155 as a student activity.

Suggested Assignment:

Work the remainder of the written exercises on page 155.

Lesson 4

Polygons

Aim:

To develop within the students an understanding of polygons.

Suggested Methods:

Discussion, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Read and discuss the material on pages 156 and 157. Develop the idea of a plane figure, a simple figure, a polygon and its parts.
3. Define a polygon.
4. Draw some polygons on the board. Name and discuss them. Point out a side, a vertex, adjacent sides, adjacent vertices, and a diagonal.
5. Work and discuss together the written exercise on page 157.

Summary:

Use the oral exercise on page 157 as a student activity.

Suggested Assignment:

Study the material that is to be used with page 157 on page 344 and answer the questions listed.

Lesson 5

Angles

Aim:

To help the student develop a better understanding of the concept of an angle as a set of points.

Suggested Methods:

Discussion, demonstration.

Suggested Materials:

Transparency of acute and obtuse angles, one of vertical angles, and one of adjacent angles

Developmental Steps:

1. Check and correct the assignment.
2. Define an angle. Use a transparency of an angle to illustrate sides, vertex, the naming of angles, the interior of an angle, the exterior of an angle. Discuss these fully.
3. Discuss the material on vertical angles and adjacent angles. Use the transparencies to illustrate them.
4. Work and discuss number 1 in the written exercise on page 159.

Summary:

Discuss the oral exercises on page 159.

Suggested Assignment:

Work the remainder of the written exercises on page 159.

Lesson 6

Measuring Angles

Aim:

To teach the students to measure angles.

Suggested Methods:

Discussion, demonstration, and drill.

Suggested Materials:

Clear plastic protractor, transparencies of acute, obtuse, right, vertical, and adjacent angles

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the meaning of 1 degree. Use the plastic protractor on the overhead projector to point out 1^{o}.
3. Define protractor.
4. Demonstrate the proper way to use a protractor. Use the protractor and some of the above transparencies to show how angles are measured.
5. Discuss the classification of angles. Use the transparencies to illustrate the classes of angles. Compare the measures of vertical angles.
6. Use the written exercise to drill the students on measuring angles.

Summary:

Discuss the oral exercises on page 161.

Suggested Assignment:

Study the material that is to be used with page 161 on page 344 and do the activities suggested.

Lesson 7

Perpendicular Lines and Planes

Aim:

To teach the students the meaning of perpendicular lines and to develop the concept of a line perpendicular to a plane.

Suggested Methods:

Discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Define perpendicular lines. Use models of perpendicular lines found in the classroom to illustrate the idea.
3. Discuss when a line is perpendicular to a plane. Use examples to illustrate this.
4. Work problems 1 and 2 in the written exercise together.

Summary:

Use the oral exercise on page 163 for class discussion.

Suggested Assignment:

Work the remainder of the problems in the written exercise on page 163.

Lesson 8

Unit Review

Aim:

To review Unit IX.

Suggested Methods:

Review, discussion, and drill.

Developmental Steps:

1. Check and correct the assignment.
2. Use pages 165-166 as a guide to review the material covered in the unit.
3. Work together examples (as many as time permits) from page 166.

Suggested Assignment:

Unit test tomorrow

Lesson 9

Test

Aim:

To test the students' ability to work with points, lines, and planes, parellel lines and planes, polygons, angles, perpendicular lines and planes.

Developmental Steps:

The Independent Test for Chapter 8 page T 166 may be used as it is.

Suggested Assignment:

None

Unit X

Measurement

Unit X

Measurement

Behavioral Objectives:

1. Using a ruler and a premeasured line segment, the student will measure that line segment to the nearest 1/4 inch, 1/8 inch, and 1/16 inch.
2. Given two (or more) measurements, the student will tell which is more precise, indicate the greatest possible error of each, and the relative error of each. He will tell which is more accurate.
3. Given measurements, the student will tell the unit of measure, the number of those units and the number of significant digits in each of the measurements.
4. Given problems involving measurements, the student will correctly add, subtract, multiply, and divide those measurements.

Lesson 1

Assumptions About Measurement

Aim:

To make the students aware of some of the basic assumptions concerning measurement that they have accepted and used during the past.

Suggested Methods:

Discussion, demonstration.

Suggested Materials:

Rulers

Developmental Steps:

1. Return and correct the tests.
2. Open the discussion about measurement by asking, "What is Measurement?" Have the students name many different kinds of measurement and the instruments used in measuring.
3. Discuss the material on page 167. Stress the three assumptions. Emphasize that to compare objects there must be a common characteristic.
4. Discuss the oral questions, 1-4, page 167.
5. Have everyone measure the line segment in number 5. Compare answers; correct mistakes.

Summary:

Restate the basic assumptions.

Suggested Assignment:

Do as directed in numbers 6-8 in the oral exercises, page 167.

Lesson 2

Precision in Measurement

Aim:

To teach the concepts of precision and greatest possible error as they pertain to measurements.

Suggested Methods:

Definition, discussion, question and answer.

Developmental Steps:

1. Check and correct the assignment.
2. Define the terms precise and precision.
3. Discuss the material on page 168 together, using illustrations.
4. Read together the statement, "The smaller the unit of measure, the greater the precision." Discuss the statement, emphasizing that this does not mean the greater the accuracy.
5. Give several examples, as 3 1/4", .05", 10". Ask the students to give the precision of each.
6. Using the last illustration on page 168, discuss the greatest possible error.
7. Read together the statement, "The greatest possible error of a measurement is equivalent to one-half the unit of measure being used." Discuss it. Emphasize that this does not refer to a mistake in measuring.
8. Discuss the small chart on page 169 and the material following it.

Summary:

Discuss the oral exercises, page 169.

Suggested Assignment:

Work the written exercises, page 169.

Lesson 3

Decimal Notation and Precision

Aim:

To extend the concept of precision to include decimal notation.

Suggested Methods:

Discussion, review.

Developmental Steps:

1. Check and correct the assignment.
2. Read together the material in the first column, page 170.
3. Work together the examples in the oral exercises, page 170.
4. Review the concept that the smaller the unit of measure, the greater the precision.
5. Compare the precision of the measurements in the oral exercises to determine which are more precise.

Summary:

Work together numbers 1 and 5 in the written exercises, page 170.

Suggested Assignment:

Work the remainder of the written exercises.

Lesson 4

Relative Error and Accuracy

Aim:

To teach the concepts of relative error and accuracy as they pertain to measurement.

Suggested Methods:

Review, discussion, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material in the first column, page 171. Use the examples 20 ft. and 4 ft. to develop the concept of relative error.
3. Discuss the idea that a relative error may be stated as a decimal or may be expressed as a per cent of error by converting it to a per cent. Review the method of changing a decimal to a per cent. Use the relative error found in step 2 to demonstrate this.
4. Discuss accuracy. Show that the smaller the relative error, the greater the accuracy. Use the example in step 2.

Summary:

1. Review the methods of determining the precision, greatest possible error and relative error.
2. Find precision, greatest possible error and relative error of measures in number one in the oral exercises, page 171.
3. Discuss number two in the oral exercises.

Suggested Assignment:

Work the written exercises, page 171.

Lesson 5

Significant Digits

Aim:

To teach the concept of significant digits and their relationship to the accuracy of a measurement.

Suggested Methods:

Discussion, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material in the first column, page 172. Use the examples of measurements to discuss significant digits.
3. Discuss the chart at the bottom of page 172.
4. Develop and discuss the concepts:
   (a) The accuracy of a measurement is independent of the unit of measure used.
   (b) The more significant digits in a numeral for a measurement, the more accurate is the measurement.
5. Discuss and give examples to illustrate the rules for determining the significant digits in a numeral. Demonstrate how to determine the number of significant digits in a fraction.

Summary:

Work together numbers 1, 6, and 9 in the written exercises.

Suggested Assignment:

Work the remainder of the written exercises, page 173.

Lesson 6

Adding and Subtracting Measures

Aim:

To teach procedures for adding and subtracting numbers that represent measures.

Suggested Methods:

Discussion, deduction.

Developmental Steps:

1. Check and correct the assignment.
2. Read and discuss together page 174.
3. Use other examples to lead the students to the deduction stated on page 175.
4. Discuss the examples in the first column, page 175.

Summary:

Answer the oral exercises, page 175. Add or subtract some of these.

Suggested Assignment:

Work the written exercises, page 175.

Lesson 7

Multiplying and Dividing Measures

Aim:

To help the students develop procedures for multiplying and dividing numbers that represent measures.

Suggested Methods:

Discussion, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material on page 176. Use an example to show the possible error involved in calculating the area of a rectangle.
3. Introduce the method of multiplying and dividing measures and rounding to the proper number of significant digits.
4. Work together some of the examples in the oral exercises, rounding correctly.

Summary:

Restate the rules for multiplying and dividing measures.

Suggested Assignment:

Work the written exercises, page 177.

Lesson 8

Problems Using Measures

Aim:

To help the students develop skill in the solving of problems involving operations on measures.

Suggested Methods:

Discussion, question and answer, review.

Developmental Steps:

1. Check and correct the assignment.
2. Have the students put down their pencils and discuss with you the problems on page 178. Discuss the accuracy of the measures involved and to what degree of accuracy the answers should be given.

Summary:

Review the procedures for operations with numbers involving measures.

Suggested Assignment:

Work problems 1-10, page 178.

Lesson 9

Chapter Review

Aim:

To review the chapter to prepare for a chapter test.

Suggested Method:

Review.

Developmental Steps:

1. Check and correct the assignment.
2. Using pages 180-181, review the chapter. Discuss the true-false statements, page 181. Work together as many examples from page 181 as time allows.

Suggested Assignment:

Chapter test tomorrow

Lesson 10

Chapter Test

Aim:

To test the students' ability to work with numbers involving measures.

Suggested Methods:

Testing.

Developmental Steps:

Test

1. At least one line segment to measure to the nearest 1/4 inch, 1/8 inch, and 1/10 inch.
2. The following parts of the Independent Test for Chapter 9:

   Part 1

   Part 2 - in addition, ask which measures are more accurate and why.

   Parts 3-7

Suggested Assignment:

None

Unit XI

Construction and Congruence

Unit XI

Construction and Congruence

Behavioral Objectives:

1. Given a line segment, the students will construct a line congruent to it.
2. The students will construct a bisector of a line segment and of an angle.
3. Giver angles and triangles, the students will construct angles and triangles congruent to those given.
4. The students will construct perpendicular lines.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Constructing Congruent Line Segments

Aim:

To show the students how to construct line segments equal in length to given line segments.

Suggested Materials:

Board compass, straightedge, student compasses

Suggested Methods:

Discussion, demonstration, illustration, definition.

Developmental Steps:

1. Return, check and discuss tests.
2. Discuss the material in the first paragraph on page 185.
3. Introduce the terms construction and straightedge. Explain their meanings, how constructions can be made, and how the straightedge is used.
4. By using a line segment $\overline{RS}$ and a working line longer than $\overline{RS}$, show how a compass is used to construct a line segment $\overline{UV}$ on the working line the same length as $\overline{RS}$.
5. Define the term congruent. Show its symbol ($\cong$). Explain the rule of congruency.
6. Have the students construct line segments according to problem 1 in the written exercise. Check their work and give assistance where needed.

Summary:

Discuss the oral exercise, page 185.

Suggested Assignment:

Do as instructed in the written exercise, page 185.

Lesson 2

Bisecting Line Segments and Angles

Aim:

To teach the students to bisect a line segment and an angle.

Supplementary Materials:

Compass, straightedge

Suggested Methods:

Illustration, discussion, demonstration, definition.

Suggested Materials:

Transparencies of a line segment bisected, and an angle to be bisected

Developmental Steps:

1. Check and correct the assignment.
2. Define the term bisect and explain it by referring to the transparency of a line segment bisected.
3. Demonstrate on the board the method of bisecting a line segment by using a compass. This could be done on a transparency using a compass with a rubber tip.
4. Introduce midpoint. Illustrate, by measuring the halves of the segment, that the point previously determined is the midpoint of the segment.
5. Have the students draw a line segment about 3 or 4 inches long and bisect it. Walk around and check their work, giving assistance where necessary.
6. Demonstrate by using an angle ($\angle PQR$) drawn on a transparency, the method of bisecting a given angle.

7. Introduce the term angle bisector. Show by using a protractor that $\overrightarrow{QT}$ is the angle bisector of $\angle PQR$.

8. Have each of the students draw an angle, and with a compass construct the bisector of the angle. Observe their operations and give assistance as necessary.

Summary:

Discuss the oral exercises, page 186.

Suggested Assignment:

Do as instructed in the written exercise, page 186,

Lesson 3

Constructing Congruent Angles

Aim:

To teach the students to construct an angle equal in size to a given angle.

Suggested Materials:

Compass, straightedge

Suggested Methods:

Demonstration, discussion, illustration.

Developmental Steps:

1. Check and correct the assignment.
2. Illustrate on the board by using a given angle ($\angle ABC$) the steps used to construct $\angle DEF$ congruent to the given angle.
3. Explain the rule of congruency of angles as stated on page 187.
4. Have the students draw angles and with their compasses and straightedge, construct angles congruent to the given angles. Give assistance to those who need it.

Summary:

Discuss the oral exercise, page 187.

Suggested Assignment:

Make the constructions described in the written exercises, page 187.

Lesson 4

Constructing Perpendicular Lines

Aim:

To teach the students to construct a line perpendicular to a given line.

Suggested Materials:

Compass, straightedge

Suggested Methods:

Demonstration, illustration, discussion, explanation.

Developmental Steps:

1. Check and correct the assignment.
2. Explain that two lines are perpendicular if they form right angles.
3. By using a line $\overleftrightarrow{PR}$, show the students how a line is constructed perpendicular to that line at a point Q on the line.

4. Introduce the perpendicular symbol. Explain how it is used.
5. Have each student draw a line and construct a perpendicular to that line at a given point on the line. Help those who need assistance.
6. Show by using a second line how a perpendicular is constructed through a point not on the line.
7. Have each student draw a line and construct a perpendicular to it from a given point not on the line. Give assistance where it is needed.

Summary:

Discuss the oral exercise, page 189.

Suggested Assignment:

Perform the indicated constructions for the written exercises, page 189. Answer the questions about them.

Lesson 5

(Supplementary Lesson)

Practice in Construction and Unit Review

Aim:

To increase the students' ability in construction.

Suggested Materials:

Compass, straightedge

Suggested Methods:

Practice, review.

Developmental Steps:

1. Check and correct the assignment.
2. Assign this lesson as follows:
   a. Assign chosen problems from those on page 188 for practice. It is suggested that you walk around while the students work and help as needed.
   b. Assign chosen problems from this lesson as additional problems with previous lessons.
3. Review each construction that has been covered in this chapter.

Summary:

This lesson is a practice exercise, so no summary is necessary.

Suggested Assignment:

Unit test

Lesson 6

Unit Test

Aim:

To test the students' ability to use a compass and straightedge in making constructions.

Suggested Materials:

Compass, straightedge

Suggested Methods:

Testing

Developmental Steps:

Give a test made up of at least one of each of the six constructions taught (constructing perpendicular lines, 2 methods) and possibly one similar to those on page 188.

Suggested Assignment:

None

Unit XII

Perimeter, Area, Volume

Unit XII

Perimeter, Area, Volume

Behavioral Objectives:

1. Given measurements of polygons, the students will state the perimeter of those polygons.
2. Given the measurement of a diameter or a radius of a circle, the students will find its circumference. (and vice-versa)
3. Given diagrams of quadrilaterals and a circle, the students will identify them.
4. Given measurements, the students will find: (a) the areas of rectangles, triangles, parallelograms, trapezoids, and circles. (b) the volumes of prisms, pyramids, cylinders, cones, and spheres.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Perimeter of a Polygon

and

Circumference of a Circle

Aim:

To reinforce the students' understanding of formulas and perimeters by considering the formulas for the perimeters of various polygons.

To develop an understanding of the circumference of a circle.

Suggested Methods:

Discussion, drill, and review.

Suggested Materials:

Transparency of a circle with the parts labeled

Developmental Steps:

1. Return and discuss tests.
2. Review the definition of a polygon.
3. Define perimeter.
4. Discuss the formula for finding the perimeter of a polygon.
5. Have the students work the written exercise on page 205.
6. Discuss the parts of a circle. A transparency of a circle with the parts labeled should be used to illustrate this. Point out the center, radius, diameter, chord, arc, and circumference of the circle.
7. Define circumference and compare it with the perimeter of a polygon.
8. Discuss the formulas for finding the circumference of a circle.
9. Work and discuss problems 1, 6, and 9 on page 207.

Summary:

Discuss the oral exercises on pages 205 and 207.

Suggested Assignment:

Work the remainder of the problems on page 207.

Lesson 2

Quadrilaterals and Area

Of a Rectangle

Aim:

To reinforce the students' understanding of a quadrilateral as a four-sided polygon. To help the students clarify their understanding of the area of a rectangle by considering the formula A = bh.

Suggested Methods:

Review, discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Review the definition of a quadrilateral.
3. Define and discuss the kinds of quadrilaterals.
4. Discuss the written exercise, page 209.
5. Define the area of a rectangle by using the formula A = bh.
6. Discuss the examples on page 210.
7. Work and discuss problem one in the written exercise together, page 210.

Summary:

Discuss the oral exercises on pages 209-210.

Suggested Assignment:

Work the written exercise on page 210.

Lesson 3

Area of a Right Triangle

Aim:

To teach the students to find the area of a right triangle.

Suggested Methods:

Review, discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Review the formula for finding the area of a rectangle.
3. Define a right triangle.
4. Read and discuss the material on page 211.
5. Work problem five together (answer according to the rules on pages 174-177.)

Summary:

Discuss the oral exercise on page 211.

Suggested Assignment:

Work the written exercises, page 211.

Lesson 4

Area of Any Triangle

Aim:

To teach the students to find the area of any triangle.

Suggested Methods:

Review, discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Review the procedure for finding the area of a right triangle.
3. Discuss the material on page 213. Develop the formula $A = 1/2\ bh$.
4. Work and discuss problems 9, 10, and 11 together.

Summary:

Discuss the oral exercise on page 213.

Suggested Assignment:

Work the remainder of the written exercise on page 213.

Lesson 5

Area of a Parallelogram

Aim:

To teach the students to find the area of a parallelogram.

Suggested Methods:

Review, discussion.

Suggested Materials:

Prepared transparency of the sketch for a parallelogram, page 214

Developmental Steps:

1. Check and correct the assignment.
2. Review the procedure for finding the area of a triangle.
3. Define the area of a parallelogram.
4. Develop the formula for finding the area of a parallelogram using a transparency of a parallelogram.
5. Work and discuss the written exercise on page 214.

Summary:

Discuss the oral exercise on page 214.

Suggested Assignment:

Work the problems on page 347 that are to be used with page 214.

Lesson 6

Area of a Trapezoid

Aim:

To teach the students to find the area of a trapezoid.

Suggested Methods:

Review, discussion.

Suggested Materials:

Prepared transparency of the sketch, page 215

Developmental Steps:

1. Check and correct the assignment.
2. Review the procedure for finding the area of a triangle.
3. Define the area of a trapezoid.
4. Use the transparency of the trapezoid to develop the formula for the area.
5. Work and discuss problem 8 in the written exercise.

Summary:

Discuss the oral exercise on page 215.

Suggested Assignment:

Work the remainder of the problems in the written exercise, page 215.

Lesson 7

Area of a Circle

Aim:

To teach the students to find the area of a circle.

Suggested Methods:

Review, discussion.

Suggested Materials:

Transparency of a circle with overlays of an inscribed octagon and an inscribed polygon with sixteen sides

Developmental Steps:

1. Check and correct the assignment.
2. Review the meanings of circumference, diameter, and radius.
3. Discuss the material on pages 216 and 217. Use the transparency and overlays to develop the formula for the area of a circle.
4. Work and discuss problem 10 in the written exercise.

Summary:

Discuss the oral exercise on page 217.

Suggested Assignment:

Complete the remainder of the written exercise on page 217.

Lesson 8

Review Finding Areas

Aim:

To review for a test on finding areas.

Suggested Methods:

Review, drill.

Developmental Steps:

1. Check and correct the assignment.
2. Review and drill the students on finding areas of: rectangles, triangles, parallelograms, trapezoids, and circles.

Summary:

Restate the formulas for finding the area of all the figures in number 2 above.

Suggested Assignment:

Study for test tomorrow.

Lesson 9

Test

Aim:

To evaluate the students' growth in finding areas.

Suggested Methods:

Test

Suggested Test:

1. Draw the quadrilaterals and a circle and have the students identify them.
2. Find the area of the following quadrilaterals:
   (a) A rectangle with a base of 35 ft. and height of 12.5 ft.
   (b) A triangle with a base of 36 yds. and an altitude of 15 yds.
   (c) A parallelogram with a base of 15 1/2 ft. and an altitude of 7 1/2 ft.
   (d) A trapezoid with a base of 28 ft. and a base of 35 ft. and an altitude of 20 ft.
   (e) A circle with a diameter of 37.4 ft.
3. Find the circumference of the circle in 2 (e).
4. Find the perimeter of the rectangle in 2 (a).

Lesson 10

Prisms

Aim:

To introduce the students to the concept of a geometric solid by a discussion of prisms.

Suggested Materials:

Models of different kinds of prisms, 1" cubes, if possible

Suggested Methods:

Discussion, demonstration, and definition.

Developmental Steps:

1. Return and correct the errors on the tests as the problems are discussed.
2. Read together and discuss the material on pages 218, including the definition of a right prism.
3. Discuss the illustrations at the top of page 218 and 219 telling what kind of prism each is.
4. Demonstrate the kinds with models (as many kinds as possible) having students name them.
5. Demonstrate with 1" cubes how to find the volume of a rectangluar solid; develop the formula V = Bh.
6. Work together to find the volumes of one or two of the models you have.

Summary:

Discuss the oral exercises, page 219.

Suggested Assignment:

Work the written exercises, page 219.

Lesson 11

Pyramids

Aim:

To introduce the students to the pyramid and develop the formula for the volume of a pyramid.

Suggested Materials:

If possible obtain hollow models of a pyramid and a rectangular solid with the same base, sand

Suggested Methods:

Discussion, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Read together and discuss the material in the first two paragraphs page 220.
3. Demonstrate with solids that three pyramids full of sand exactly fills the rectangular solid. Emphasize that the bases are the same.
4. Develop the formula $V = 1/3\ Bh$.
5. Work together to find the volume of your model pyramid.

Summary:

Review the formulas for the areas of a square and a triangle, and the volume of a pyramid.

Suggested Assignment:

Work the written exercises, page 220.

Lesson 12

Area of a Cylinder

Aim:

To introduce the circular cylinder and develop procedures for finding the surface area of a cylinder.

Suggested Materials:

Models of cylinders

Suggested Methods:

Discussion and demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Show model (s) of cylinders. Define right circular cylinder.
3. Using a paper model, show that the lateral surface is actually the same as a rectangle with the circumference of the base of the cylinder equal to the length of the rectangle and the altitude of the cylinder equal to the width of the rectangle.
4. Show that the lateral area plus the areas of the two bases is equal to the total surface area. (It is not necessary to use the formula presented, unless in the first form only.)

Summary:

Discuss the oral exercise, page 221.

Suggested Assignment:

Work the written exercises, page 221.

Lesson 13

Volume of a Cylinder

Aim.

To help the students determine and use the formula of the volume of a right circular cylinder.

Suggested Methods:

Discussion, demonstration, review, practice.

Developmental Steps:

1. Check and correct the assignment.
2. Read and discuss the material on page 222.
3. Using the example at the top of the first column, page 223, work out volume and surface area. (Review circumference.)
4. Discuss together the oral exercises, page 223.

Summary:

Work together problems numbered 4 and 9.

Suggested Assignment:

Work the remainder of the written exercises. (Number 10 could be made optional.)

Lesson 14

Volume of a Cone

Aim:

To teach the students to determine and use the formula for the volume of a cone.

Suggested Methods:

Discussion, review, demonstration.

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the material in the first column, page 224.
3. Recall the formula for the volume of a pyramid as compared to that of the rectangular solid. Show that a cone is related to a cylinder in the same way.
4. Demonstrate how to find the volumes of several cones with diameter or radius, and height given as:

   radius 2.4, h 5.8, volume = _____

   diameter 3 1/4, h 6 1/2, volume = _____

Summary:

Review the formula for the volume of a cone.

Suggested Assignment:

Work the written exercises, page 224.

Lesson 15

Area and Volume of a Sphere

Aim:

To teach the students the formulas for the area and volume of a sphere.

Suggested Methods:

Discussion, demonstration.

Suggested Materials:

Prepared transparency of the sketches, page 226

Developmental Steps:

1. Check and correct the assignment.
2. Discuss the sphere. Use the transparency as you define "great circle" and "small circle" of a sphere.
3. Levelop the formula for the area of a sphere. Use the example on page 226 to find the surface area of a sphere.
4. Introduce the formula for the volume of a sphere. Use an example to find the volume of a sphere. Encourage student participation in finding the volume.

Summary:

Discuss the oral exercises, page 227, and numbers 1 and 6 in the written exercises.

Suggested Assignment:

Work the written exercises, page 227.

Lesson 16

Review

Aim:

To review the chapter to prepare for a chapter test.

Suggested Methods:

Review

Developmental Steps:

1. Check and correct the assignment.
2. Using pages 229-230 as a guide, review the chapter, working as many of the problems as time permits.

Suggested Assignment:

Chapter test.

Lesson 17

Chapter Test

Aim:

To test the students' behavior as stated in the behavioral objectives for Unit XII.

Suggested Methods:

Test

Developmental Steps:

1. Give drawings as those in part 1, page 230 (except 5th). Have the students name the figures and find the areas of the figures in 1 and 2, and the surface areas and volumes of those in 3-5.
2. One or two problems similar to problem 2, part 2, page 230.

Unit XIII

Scientific Notation and the Metric System

Unit XIII

Scientific Notation and the Metric System

Behavioral Objectives:

1. Given linear metric measurements, the student will convert them to other linear metric measures.
2. Given standard measures, the student will convert them to metric measures.

Degree:

The degree to which the student performs the above objectives will be determined by the teacher. An acceptable degree might be successful performance 75% of the time when the student is confronted with such problems.

Lesson 1

Linear Metric Measure

Aim:

To teach the students the relationship between metric and English linear measure and how to convert one to the other.

Suggested Methods:

Demonstration, explanation, review.

Developmental Steps:

1. Check and correct the test.
2. It should be explained that the metric system of measure is based on ten and powers of ten, so therefore, is more convenient to scientists than is the English system.
3. Introduce the meter, the basic unit of linear measure, and explain that as it was originally defined, a meter was one ten-millionth of the distance from the North Pole to the equator along a meridian. In terms of the English system, a meter is a little longer than one yard. (Introduce the equivalency formula.) (1 meter = 39.37 inches, approximately)
4. Show the class the following equivalency chart, and emphasize that each metric unit in the first column uses the word meter with a prefix.

Name of Unit	Abbreviation	Equivalent in Meters
1 Millimeter	1 mm.	1/1000 or .001
1 centimeter	1 cm.	1/1000 or .01
1 decimeter	1 dm.	1/10 or .1
1 meter	1 m	1 m.
1 dekameter	1 dkm.	10 m.
1 hectometer	1 hm	100 m.
1 kilometer	1 km	1000 m.

Explain that milli means 1/1000, centi means 1/100, deci means 1/10 and so on.

5. Emphasize that each increasing or each decreasing unit is ten times as large or 1/10 as large as the one before it.
6. Introduce the term mega (meter), explaining its meaning and stating that it has become increasingly more useful in recent years. Explain also that the micron is used rather than micrometer. Mega means one million, micro means one-millionth. Add these terms to the above chart. (Use the symbol for micron, μ.)
7. Using the examples on page 291, demonstrate how metric measure is converted to other metric measures, and to English measure. Use an example to illustrate how English measure is converted to metric measure.

Summary:

Discuss the oral exercise, page 291.

Suggested Assignment:

Work the written exercise, page 291.

Lesson 2

Unit Review

Aim:

To review the lesson on linear metric measure in preparation for a test.

Suggested Methods:

Review, discussion.

Developmental Steps:

1. Check and correct the assignment.
2. Review the conversions as studied.

Suggested Assignment:

Unit test.

Lesson 3

Unit Test

Aim:

To test the students' ability to convert metric measure to other metric measure, metric measure to English measure, and English measure to metric measure.

Suggested Methods:

Testing

Developmental Steps:

Give a test consisting of problems like those in the written exercises on page 291 and the "Self Evaluation" exercises 3-7, Part 3, page 298.

Suggested Assignment:

None